Centripetal Force

Purpose

The purpose of this experiment is to empirically determine the relationship between the angular velocity of a body in circular motion and:

- the centripetal force F necessary to maintain a constant angular velocity
- the mass M of the body and
- the radius R of the circular path.

Equipment

Centripetal force apparatus with cylindrical bob; set of four different springs; three-spring extenders; slotted weights; weight hanger; string, log-log graph paper.

Preliminary Discussion

In this experiment we will study the motion of a body which is moving in a circle at a constant speed. We will try to deduce a mathematical expression from which you could predict the behavior of any body moving in a circle under the influence of a centripetal force, by conducting a systematic set of experiments and using graphical and computer methods to analyze the results.

The parameters which we will be measuring are the mass M and angular velocity ω of the body, the radius R of the circular path, and the force F necessary to keep the body moving in a circular path when the speed is constant. This force is directed towards the center of the circular path and is called a centripetal force. The angular velocity, ω, for circular motion is related to the speed v of the object through the relation, $\omega = v/R$.

We will determine the functional dependence of the angular velocity ω on each of the other parameters by performing three experiments. In each experiment one of the parameters will be varied while the other two are held constant, i.e., we will measure ω as a function of:

1. the centripetal force, F, holding the mass and radius constant;
2. the mass, M, holding the force and radius constant;
3. the radius, R, holding the force and mass constant.

For each experiment, we will assume that the functional dependence of the angular velocity on the varied parameter may be written in the form

$$\omega = aX^n$$

where X is the varied parameter (either F, M or R). This is a power law relationship between the varied parameters (F, M or R) and the angular velocity ω. The value of the exponent, *n*, will then be determined experimentally by graphing the data obtained for each experiment A, B and C.

How will the exponent n be determined? Note that if we take the logarithm of the power law equation $\omega = aX^n$, we get

$$log\ \omega = nloga + nlogX$$

where nloga is a constant term. Hence,

$$\log \omega = nlogX + c.$$

This equation is the linear form y = mx + b. Therefore if log ω is plotted against log$X$ where X is either F, M or R, the result is a straight line with the slope being the appropriate exponent n.

From the graphs in Experiments A, B and C, we will have obtained three exponents which we will label a, b, and g, respectively. We can then write our final result for the angular velocity in the form

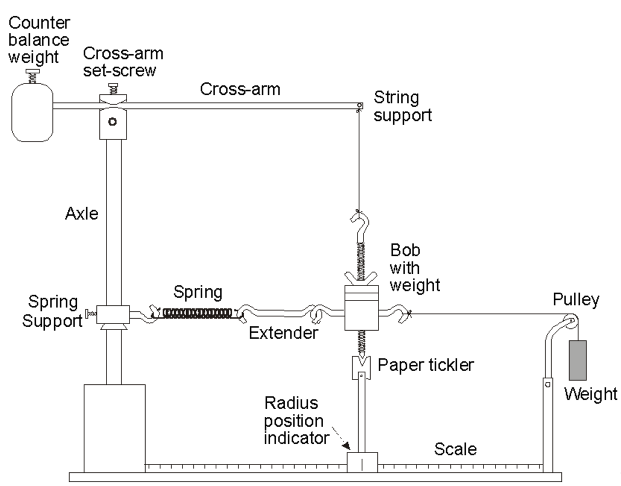
$$\omega = kF^aM^bR^g$$

NOTE: k is a constant which we will not determine in this lab.

Experimental Procedure

The apparatus which will be used for this lab is depicted in the figure. A cylindrical bob is suspended by means of a string support from a cross-arm. The cross-arm is in turn supported by an axle which is free to rotate. The position of the bob relative to the center of rotation may be changed by loosening the cross-arm about its support. The centripetal force is provided by a spring support on the axle. The vertical position of the spring support should be adjusted such that its hook is at the same height as the hook on the bob when the bob is hanging vertically. A position indicator consisting of a rod with a spring clip is used to support a narrow piece of paper. The height of the bob should be adjusted such that the tip of bob is slightly below the upper edge of the paper ticker when the bob is hanging vertically.

A. Measuring the angular velocity as a function of centripetal force (mass and radius constant).



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Secure two one-hundred gram weights on the bob with their slots pointing away from the center of rotation. Set the position indicator to a radius between 20 and 25 cm. *Record the constant mass (mass of the bob by itself + added slotted masses) and the constant radius used for this experiment.* Move the cross-arm so that the bob pointer touches the center of the paper ticker. Use the counter balance weight to balance the cross-arm. Tighten the counterbalance and cross-arm set screws. Hook a spring to the bob and axle (you may need to use a spring extender). Next rotate the axle clockwise as viewed from the top. With a little practice you will be able to rotate the axle at a constant angular velocity such that the bob hangs vertically. The pointed tip of the bob should now be ticking the paper on the position indicator. Using a stopwatch, obtain the time t for at least 10 full revolutions of the bob. This value will be used to determine the angular velocity of the bob ω = v/R, where v = (2πR*10)/t. With the system at rest, hang weights (using a holder attached by a string which passes over the pulley and attaches to the bob) until the string is stretched enough for the bob to hang vertically again. Adjust the height of the pulley until the string is level. This allows you to determine the centripetal force which was present when you were rotating the bob. Record the time for 10 revolutions and the centripetal force. *Repeat the above procedure with different spring and extender combinations until you have at least five different combinations of ω and F.* Remember to get as wide a range of force values as possible to reduce the error in your final result.

B. Measuring the angular velocity as a function of mass (force and radius constant):

Use the radius setting and one of the spring/extender combinations (preferable one that gives a centripetal force near the middle of the force range) from Experiment A as the constant radius and constant centripetal force to be used for this experiment. Record these two values. Start with the bob by itself. Rotate the axle and obtain the time t for 10 revolutions just like you did in Experiment A. Record the mass *M* used and the time t. Change the mass of the bob using slotted weights and find the dependence of the angular velocity ω on the mass *M*. (*You should use at least five different masses*.)

C. Measuring the angular velocity as a function of radius (force and mass constant):

Use the same bob mass in Experiment A as the constant mass. Use the spring/extender combination from Experiment B and hang sufficient weights over the pulley from the bob to bring the bob to a vertical position to determine the centripetal force. This is the constant force to be used for this part. Record the values for the constant mass and the constant force. Now remove the weight from the pulley and bob and rotate the axle as before and obtain the time t for 10 revolution. Record the radius *R* and time t. Next, to get a different value for *R*, change the spring/extender combination and hang the constant force on the pulley and bob. Readjust the cross-arm so that the bob hangs vertically and balance the cross-arm. Note the new value for *R*. Remove the weight from the pulley and bob and rotate the axle as before, timing 10 revolutions and determining the angular velocity for the new radius. Repeat this procedure, keeping the mass of the bob and the weight hanging over the pulley the same, *until you have angular velocities for five different radii*. Find the dependence of the angular velocity on the radius.

Data Analysis

Using Excel, enter the data obtained for Experiment A. Put a title to identify this data table and also fill in your name and partners' names. Make sure to identify the constant values used (constant R, constant M) and the two variables (Centripetal force F and time t) observed and recorded for this case. Calculate the angular velocity ω. Create a column for *log(F)* and generate its values using the function *=log(cell# for the value of F, 10)*. Similarly, create a column for *log(ω).*

Perform a least squares fit plot for the logarithm of the angular velocity *log(ω)* versus the logarithm of the force *log(F)* using Excel's *"DATA" – "Data Analysis" – "Regression"* feature. Use *log(ω)* for the Y-Range and *log(F)* for the X-Range. From the graph, identify the slope and the error in the slope and record it in your data table.

Print out the resulting graph and its accompanying tables of values. Also, print out the data table you made for Experiment A.

Repeat the above for *log(ω) vs log(M)* (Experiment B) and *log ω vs log(R)* (Experiment C).

Summary of Results

As a minimum, your report should include all your data, all the plots and your best values of a, b and g and their estimated errors. Also compare your results with the expected results. To make the comparison with the expected results, write your results from Experiments A, B and C in the form

$$\omega = kF^a M^b R^g$$

Then rewrite your results so they appear in a more familiar form? (HINT: Solve for the centripetal force F, as a function of ω, M, and R.) From this form you should be able to determine the expected values of the exponents. What values would you expect to find for the constant k?