Escape Velocity

Purpose

To study the use of computer simulations for the solution of problems. We will use simulation software to answer the question: what is the minimum velocity of a projectile necessary to escape the pull of the Earth's gravity.

Apparatus

Orbits simulation software, spreadsheet program

I. Preliminary Discussion

Until recently, physicists could study nature only by doing direct experiments and then develop mathematical models which could describe the results of the experiments and predict the results of experiments not yet done. With the advent of faster and less expensive computers, it has become possible to simulate experiments that would be either impossible or impractical to carry out. Computer simulations are not a substitute for true experiments or theory in discovering new phenomena, but are nonetheless valuable tools. In this experiment we will use a simple simulation program to explore the gravitational pull between two bodies.

One of the most interesting and important phenomena which occurs in nature is the motion of objects in a field (gravitational, electric, magnetic, etc). The motions of the planets around the sun, of satellites around planets, and of electrons around a nucleus are common examples. In each of these cases, the force is an attractive one and inversely proportional to the square of the distance between the two objects. The force due to gravity, $F$, obeys the following relation:

$$ F = G \frac{Mm}{r^2} $$

where $G$ is the universal gravitational constant ($6.67 \times 10^{-11} \frac{N \cdot m^2}{kg^2}$), $M$ and $m$ are the masses of the two bodies, and $r$ is the distance between the centers of the two bodies.

If a projectile is fired vertically upward from the surface of the earth, it will travel in a straight line. If the velocity is less than the escape velocity, the projectile will eventually stop and fall back to earth. If the velocity is greater than the escape velocity, the projectile will keep going and not return.

The simplest way to calculate the escape velocity is through conservation of energy. The total energy ($E$) of a system is simply the sum of its kinetic and potential energies ($T$ and $U$ respectively). The kinetic and potential energies can be calculated using the following formulas:

$$ T_p = \frac{1}{2} mv^2, T_E = \frac{1}{2} Mv^2, U = -G \frac{Mm}{r} $$

where $T_p$ and $m$ are the kinetic energy and mass of the projectile, $T_E$ and $M$ are the kinetic energy
and mass of the Earth. Because the mass of the Earth is very much larger than that of the projectile, we can assume the Earth to be stationary, i.e. $T_E = 0$. Therefore, our equation for the total energy of the system (boils down to) can be written as:

$$E_T = T_p + U = \frac{1}{2}mv^2 - G\frac{Mm}{r}$$

Since the total energy of the system remains constant, we need only look at the value of $E_T$. If $E_T$ is negative, i.e. the potential energy is larger than the kinetic energy, the projectile will return to Earth. $E_T$ can be changed by changing the speed of the particle, $v$, and thus the kinetic energy term. If $E_T$ is positive, the projectile will continue in a straight line and not return to Earth. The velocity at which the total energy is zero is what we call the escape velocity. We will determine the escape velocity for a sample projectile using the Orbit Xplorer 2.2 computer simulation.

II. Experiment

A. Escape Velocity

**Experimental Procedure**

The simulation experiment involves launching a rocket with an upward velocity (Y Velocity) $v_y$ and monitoring the total energy of the earth-rocket system. The simulation can be used to verify whether the rocket falls back to earth or escapes the earth’s gravitational field and never return to earth. The obtained data on $v_y$ and the total energy $E_T$ will be used to determine the escape velocity of the rocket.

- Run the simulation program “Orbit Xplorer 2.2”. Start the rocket launch simulation using File → Open and select Rocket launch.sim.

  You will see a screen displaying the parameters such as mass, radius, x, y and y positions, x, y and z velocities of the earth and the rocket which will be launched form the earth.

- Start the simulation exercise by launching the rocket with a $v_y$ of 9000 m/s. Enter this value “9000” (equivalent to 9.0 km/s) in the box for the $v_y$ for the rocket. **NOTE: Make sure that you do not make any changes on the existing values of the other parameters for the earth and the rocket throughout the entire experiment.**

- Upon hitting return, the main simulation screen appears. Click on “View” - “Energy” to see the value of the Total Energy of the system on the upper right hand side of the screen. **Record both the Y velocity $v_y$ in km/s (to convert ms/ to km/s divide by 1000) and the corresponding total energy $E_T$.** While on this simulation screen hit the “Start” to see if the rocket falls back to earth or flies off. You can speed up the simulation display by clicking on the red “up arrow” on the upper right hand side. (The rocket takes several seconds to reach its maximum height.)

- Click on “Parameters” to go back to the parameter screen. Increase the Y velocity $v_y$ of the projectile by 1000 m/s and record the corresponding $E_T$ as in the previous step. Continue to increase the $v_y$ by 1000 m/s up to 15000 m/s. Record all your $v_y$ values (in km/s) and the total energy values. At this point you may not want to click on the “Start” button on the simulation screen since it can take up to a minute for the
Experimental Analysis

The last $v_y$ velocity you used is very close to the escape velocity. We can interpolate a more exact solution by graphing our data.

- Enter your data into an Excel spreadsheet. Calculate and create a column for $v_y^2$ and position this column right next to the column for $E_T$.
- Using the graphing functions, construct a plot of $E_T$ (x-axis) vs $v_y^2$ (y-axis). Select “x y scatter” for chart type.
- Get the best fit line and the linear equation ($y = mx + b$) for the graph using the ‘Trendline’ option on Excel. Select ‘Linear’ fit and ‘Display Equation on Chart’.
- Label your graph. Put your name and lab partner’s name on the graph. Then print a copy for each member of the group.
- Rewrite the linear equation obtained from Excel by substituting $v_y^2$ for $y$ and $E_T$ for $x$ which are the actual variables that you plotted. What is the significance of the $y$-intercept?
- Calculate the escape velocity from the above equation. Note that the velocity $v_y$ at which the total energy is zero ($E_T = 0$) is what we call the escape velocity.

III. Discussion Questions

1. What is the escape velocity of the projectile, based on your graphical analysis?

2. Using the equation for total energy, calculate the escape velocity of a projectile using the following numbers: (Hint: what is $E_T$ when $v$ is exactly the escape velocity?)

$$E_T = \frac{1}{2}mv^2 - G\frac{Mm}{r}$$

$$G = 6.67 \times 10^{-11} \frac{N \cdot m^2}{kg^2}, M = 5.979 \times 10^{24} kg, m = 1 \times 10^7 kg, r = 6371 \times 10^3 m$$

$M$ and $r$ are the mass and radius of the Earth respectively, and $m$ the mass of the projectile.

3. Based on the escape velocity you found today, what would the escape velocity be of a projectile that had twice the mass? Half the mass? (Hint: in Q2 above, what happens to “little” $m$ in the equation?)