Escape Velocity

Purpose

To study the use of computer simulations for the solution of problems. We will use simulation software to answer the question: what is the minimum velocity of a projectile necessary to escape the pull of the Earth's gravity.

Apparatus

Orbits simulation software, spreadsheet program

I. Preliminary Discussion

Until recently, physicists could study nature only by doing direct experiments and then develop mathematical models which could describe the results of the experiments and predict the results of experiments not yet done. With the advent of faster and less expensive computers, it has become possible to simulate experiments that would be either impossible or impractical to carry out. Computer simulations are not a substitute for true experiments or theory in discovering new phenomena, but are nonetheless valuable tools. In this experiment we will use a simple simulation program to explore the gravitational pull between two bodies.

One of the most interesting and important phenomena which occurs in nature is the motion of objects in a field (gravitational, electric, magnetic, etc). The motions of the planets around the sun, of satellites around planets, and of electrons around a nucleus are common examples. In each of these cases, the force is an attractive one and inversely proportional to the square of the distance between the two objects. The force due to gravity, *F,* obeys the following relation:

$$F = G\frac{Mm}{r^2}$$

where G is the universal gravitational constant ($6.67\times10^{-11}\,\frac{N\cdot m^2}{kg^2}$), M and m are the masses of the two bodies, and r is the distance between the centers of the two bodies.

If a projectile is fired vertically upward from the surface of the earth, it will travel in a straight line. If the velocity is less than the escape velocity, the projectile will eventually stop and fall back to earth. If the velocity is greater than the escape velocity, the projectile will keep going and not return.

The simplest way to calculate the escape velocity is through conservation of energy. The total energy *(E)* of a system is simply the sum of its kinetic and potential energies (T and U respectively). The kinetic and potential energies can be calculated using the following formulas:

$$T_p = \frac{1}{2}mv^2, T_E = \frac{1}{2}Mv^2, U = -G\frac{Mm}{r}$$

where T_p and m are the kinetic energy and mass of the projectile, T_E and M are the kinetic energy and mass of the Earth. Because the mass of the Earth is very much larger than that of the projectile, we *can* assume the Earth to be stationary, i.e. $T_E = O$. Therefore, our equation for the total energy of the system (boils down to) *can* be written as:

$$E_T = T_p + U = \frac{1}{2}mv^2 - G\frac{Mm}{r}$$

Since the total energy of the system remains constant, we need only look at the value of E_T. If E_T is negative, i.e. the potential energy is larger than the kinetic energy, the projectile will return to Earth. E_T can be changed by changing the speed of the particle, *v,* and thus the kinetic energy term. If E_T is positive, the projectile will continue in a straight line and not return to Earth. The velocity at which the total energy is zero is what we call the escape velocity. We will determine the escape velocity for a sample projectile using the Orbit Xplorer 2.2 computer simulation.

II. Experiment

A. Escape Velocity

Experimental Procedure

The simulation experiment involves launching a rocket with an upward velocity (Y Velocity) v_y and monitoring the total energy of the earth-rocket system. The simulation can be used to verify whether the rocket falls back to earth or escapes the earth's gravitational field and never return to earth. The obtained data on v_y and the total energy E_T will be used to determine the escape velocity of the rocket.

- Run the simulation program "Orbit Xplorer 2.2". Start the rocket launch simulation using File → Open and select Rocket launch.sim.

You will *see* a screen displaying the parameters such as mass, radius, x, y and y positions, x, y and z velocities of the earth and the rocket which will be launched form the earth.

- Start the simulation exercise by launching the rocket with a v_y of 9000 m/s. Enter this value "9000" (equivalent to 9.0 km/s) in the box for the v_y for the rocket. **NOTE: Make sure that you do not make any changes on the existing values of the other parameters for the earth and the rocket throughout the entire experiment.**
- Upon hitting return, the main simulation screen appears. Click on "View" - "Energy" to see the value of the Total Energy of the system on the upper right hand side of the screen. **Record both the Y velocity v_y in km/s (to convert ms/ to km/s divide by 1000) and the corresponding total energy E_T.** While on this simulation screen hit the "Start" to see if the rocket falls back to earth or flies off. You can speed up the simulation display by clicking on the red "up arrow" on the upper right hand side. (The rocket takes several seconds to reach its maximum height.)
- Click on "Parameters" to go back to the parameter screen. Increase the Y velocity v_y of the projectile by 1000 m/s and record the corresponding E_T as in the previous step. Continue to increase the v_y by 1000 m/s up to 15000 m/s. Record all your v_y values (in km/s) and the total energy values. At this point you may not want to click on the "Start" button on the simulation screen since it can take up to a minute for the

simulation to complete.

Experimental Analysis

The last v_y velocity you used is very close to the escape velocity. We can interpolate a more exact solution by graphing our data.

- Enter your data into an Excel spreadsheet. Calculate and create a column for v_y^2 and position this column right next to the column for E_T.
- Using the graphing functions, construct a plot of E_T (x-axis) vs v_y^2 (y-axis) Select "x y scatter" for chart type.
- Get the best fit line and the linear equation $(y = mx + b)$ for the graph using the '*Trendline*' option on Excel. Select *'Linear'* fit and *'Display Equation on Chart'*.
- Label your graph. Put your name and lab partner's name on the graph. Then print a copy for each member of the group.
- Rewrite the linear equation obtained from Excel by substituting v_y^2 for y and E_T for x which are the actual variables that you plotted. What is the significance of the y-intercept?
- Calculate the escape velocity from the above equation. Note that the velocity v_y at which the total energy is zero ($E_T = 0$) is what we call the escape velocity.

III. Discussion Questions

1. What is the escape velocity of the projectile, based on your graphical analysis?

2. Using the equation for total energy, calculate the escape velocity of a projectile using the following numbers: (Hint: what is E_T when v is exactly the escape velocity?)

$$E_T = \frac{1}{2}mv^2 - G\frac{Mm}{r}$$

$$G = 6.67 \times 10^{-11}\frac{N \cdot m^2}{kg^2}, M = 5.979 \times 10^{24}\,kg, m = 1 \times 10^7\,kg, r = 6371 \times 10^3\,m$$

M and r are the mass and radius of the Earth respectively, and m the mass of the projectile.

3. Based on the escape velocity you found today, what would the escape velocity be of a projectile that had twice the mass? Half the mass? (Hint: in Q2 above, what happens to "little" m in the equation?)