Hooke's Law and Simple Harmonic Motion

Introduction

A periodic motion is one that repeats itself in successive equal intervals of time, the time required for one complete repetition of the motion being called its period. Imagine, for example, a particle moving back and forth along a straight line between two fixed points. If the particle moves in such a way that its acceleration is always proportional to its displacement from the midpoint of its path, and is always directed toward that midpoint, then the motion is said to be simple harmonic.

If a mass undergoes simple harmonic motion, the force acting on it must be one which varies in just the way that the acceleration is to vary; that is, the mass must be acted upon by a force that is proportional to the displacement of the mass from its center or equilibrium position and directed toward that position. One can, without undue complication, apply a force that varies in this way, and thus cause the mass to execute a simple harmonic motion.

Suppose, for instance, that the mass is suspended from a spring. If, when the mass is above or below its equilibrium position, there exists a restoring force proportional to the displacement from equilibrium (and, according to Hooke's law such should be the case), then just the type of force is present that is required to produce simple harmonic motion.

For a Hooke's law restoring force, the relationship between the force and the displacement is given by $F = -ky$ where k is called the force (spring) constant. Application of such a force to a mass m yields

$F = -ky = ma$ or $a = -y*(k/m)$

which is the mathematical statement of the condition for simple harmonic motion discussed in the first paragraph. When $y = 0$ the mass is at the center or equilibrium position. Note that the restoring force must exist for both positive and negative y. In your text it is shown that the resultant equation of motion of the mass (variation of y with time) is given by

$y = A\sin(2\pi t/T)$ where $T = 2\pi\sqrt{m/k}$ is the period, A is the amplitude of the motion,

and t is the time elapsed starting from $y = 0$ at $t = 0$.

Experiment:

In this experiment your two major tasks will be:

1. To determine the force constant, k, of a given spring by application of Hooke's law.

2. To show experimentally that the period, T, of the motion of a mass, m, hanging from a spring depends on m and k in the manner given by the equation above for the case of simple harmonic motion.

Apparatus:

Electrical timer, set of slotted weights, hanger, spring, string, meter stick, table clamp, rod and support. You will also want to make use of the computer for this laboratory.

Procedure:

1. Hang the spring with the large end down. Place a 50 *gram* mass hanger on the spring. Record the equilibrium position of the hanger. Apply a series of forces to the spring by hanging weights from about 1 N to 6 N. Plot force added to the hanger versus displacement of the hanger from equilibrium and determine the force constant k of the spring from your graph.
2. Determine the period of oscillation for at least 5 different masses added to your spring. It will be convenient now to include the mass of the hanger as part of the added mass. Time at least 20 oscillations for each load and repeat. Keep the range of the loads which you use as large as is practical.
3. Plot T^2 versus the mass on the spring. Begin your horizontal added mass scale at -0.1 *kg* rather than 0.0 *kg* to avoid having to replot your graph to do the analysis below.

Analysis of the period versus mass measurements:

Since $T^2 = (4\pi^2 m/k)$ for a simple harmonic oscillator, what sort of graph would be obtained when plotting T^2 versus m for such a system? What would be the slope of such a plot? Does your plot yield a similar result? Compare the slope of your plot with the slope you calculate assuming simple harmonic motion and using your measured value of k from Part 1. Are they the same within the accuracy of your measurements?

So far it has been assumed that the mass of the spring is negligible. If such were the case, would your graph pass through the origin. If such is not the case, at what added mass, m, does $T = 0$ correspond to on your graph? To interpret this result, consider the mass m in the equation $T^2 = (4\pi^2 m/k)$ to be the total mass in motion including some contribution from the spring. Then $m = m_a + m_e$, where m_a is the mass added to the spring and m_e is that part of the spring mass which is effectively part of the load. Assume m_e is constant and rewrite the equation for T^2 in terms of m_a and m_e. From a graph of T^2 versus m_a determine m_e. Compare your measured m_e with the total mass of the spring. Explain why it is less than the total mass.

For simple harmonic motion, the period does not depend upon the amplitude of the oscillations. If time permits perform a brief experimental check of this prediction.

Instructions for Computer Analysis

The use of the computer for this lab is very straightforward so you will not need too many instructions.

Fill in your name, the date and your section a page

Fill in the data for displacement from equilibrium versus force.

Fill in the data for period versus the mass of the oscillating bob.

Analyze and graph your data.

Print the results of the fits to your force versus displacement data and your period versus mass data. Use these results to answer the questions in the lab write-up.

Include your data, results and graphs in your report.