Static Equilibrium of Rigid Bodies: Torques

Purpose

1. To study the relationship between force, lever arm, and torque and to see how torques add.

2. To study the conditions for static equilibrium of rigid body subject to two or more forces applied at different points of the body.

3. To study what "center of gravity" means.

Apparatus

The apparatus as shown below consists of a meter stick suspended from a fulcrum or pivot that may be moved along the length of the stick. Weights may be hung from the meter stick at various positions along the stick by means of clamps in order to apply torques.

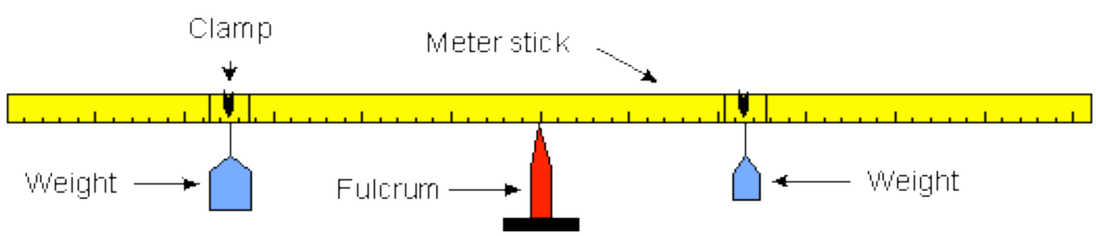

Figure 6.1 - Arrangement for balancing torques.

Procedure

First, adjust the location of the fulcrum so that the meter stick, with no weights hanging on it, is in static equilibrium (balance) in a horizontal position. Then with a clamp, hang a 200 gram mass on the meter stick about 20 cm from the fulcrum. Now hang a second 200 gram mass on the opposite arm of the meter stick in such a position that the meter stick is again in equilibrium in a horizontal position. Record the position of the fulcrum, the masses added to each side of the stick and their locations to at least the nearest millimeter. Repeat this procedure changing the second mass successively to values such as 500 gram, 400 gram, 300 gram, 250 gram, 150 gram and 100 gram. In each case adjust the position of the second mass for balance and record the data. *Be sure to weigh the clamps and include their mass in the added mass in all cases.*

As you learned in your study of statics, when the meter stick is in equilibrium there can be no net torque τ on the stick about any axis. Otherwise the stick would undergo rotation about that axis with an angular acceleration α given by $\tau_{net} = I\alpha$ where I is the moment of inertia of the stick about the axis in question.

Torques may be represented by vectors drawn perpendicular to the plane determined by the force and its lever arm. (The lever arm of a force about an axis is the perpendicular distance between the line of action of the force and the axis.) The magnitude of the torque vector is equal to the product of the force and its lever arm, while the sense of the vector is the direction of advance of a righthanded screw being turned by the torque. The net torque about an axis is then the vector sum of the individual torques about that axis.

When the meter stick alone is in equilibrium there are two forces acting on it, the force of gravity downward and the force of the fulcrum upward. Since the force of the fulcrum produces no torque about the balance axis it follows that the force of gravity also can produce no net torque about that axis. Hence, it must be that the effect of the distributed weight of the meter stick on opposite sides of the fulcrum is to produce equal and opposite torques about the fulcrum. The effect is the same as if all of the weight of the stick were concentrated at the fulcrum. The fulcrum is then said to be at the center of gravity of the stick. Hence, as long as the fulcrum is kept at the center of gravity of the meter stick while weights are added to each side, only the torques produced by the added weights need be considered.

Since you have kept the added torque on one side of the meter stick fixed, the torque produced by any weight W added to the other side must be equal in magnitude and opposite in sense to the fixed torque if the stick is to remain in balance at its center of gravity.

To see if your experimental data support such a conclusion, you will plot the product WL versus W for all the weights used to balance the fixed torque where L is the lever arm for each weight W. Do this on the computer using Excel. First prepare a data table with the weights W (including clamps) in Newtons in one column and the products WL in Newton-meters in the next column.

Use Excel to graph WL vs W using "Linear Regression Fit". The weights W should be on the x-axis and WL is on the y-axis.

Print out your graph and data table, make copies for each student in your group.

Are your results consistent with the statement that "any combination of W and L on one side of the fulcrum that produces a torque WL equal and opposite the fixed torque on the other side of the fulcrum results in equilibrium"? What value of the slope do you obtain from your computer analysis for the average of your values of WL? What is the +/- uncertainty in the average? The slope should be zero within the error of the slope. Check and make sure that your slope is zero within uncertainty. How does this average compare with the fixed torque on the other side of the stick?

Question: *How does the force necessary to balance a fixed torque depend upon its lever arm?*

To investigate this question, make a column in Excel for the reciprocal of the lever arm, 1/L. Use the Linear Regression in Excel to make a graph of the added weights, W, versus the reciprocals of their respective lever arms (1/L). Your "best-fit" line should pass through the origin.

Print copies of this graph and data table for each student in the group. Analyze your graph and data and answer the question above. What does the slope of this graph indicate?

Question: *How is the resultant of several torques determined?*

To answer this question, use 3 weights on the meter stick: Maintain the fulcrum at the center of gravity as before and balance the meter stick with a single weight on one arm and two weights at different places on the opposite arm. Record the data, calculate the magnitude of the various torques and find their vector sum. Draw a diagram such that the torques may be labeled as + (into the paper) and - (out from the paper). Does your vector sum have the value you expect (within experimental error)?

Question: *Does all of the weight of the meter stick still appear to act at the center of gravity even when the fulcrum and the center of gravity are displaced from one another.*

So far in this experiment the fulcrum has been placed only at the center of gravity of the meter stick. Suppose now that the fulcrum is placed off to one side of the center of gravity. Can all of the weight of the meter stick still be considered to be at the center of gravity as far as the torque produced by gravity is concerned? To see if this is the case, move the fulcrum to a point between the 10 cm and 25 cm marks. Balance the meter stick by applying a single mass between 100 and 500 grams to the shorter side. Record this arrangement and then weigh the meter stick. Using the condition for equilibrium, calculate the distance from the fulcrum to the point on the meter stick where all of the weight of the stick appears to act. Compare this point on the stick with the previously determined center of gravity.

The complete set of conditions for static equilibrium of a rigid body also includes the fact that the net force on the body must be zero. (Otherwise the center of mass would accelerate).

Draw a diagram for your *last* experimental situation and draw vectors for *all* of the forces acting on the meter stick. What must be the force in Newtons exerted by the *fulcrum on the meter stick* for this particular set-up when the system is in equilibrium? Indicate how you calculate this. From your knowledge of all of the forces acting on the meter stick, you can now calculate the net torque about any axis. Take the 0 cm end of the meter stick and calculate the net torque about it in the case of your last experimental set-up. What should the result be? Is this the case within experimental error?