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Varying-G cosmology with type Ia supernovae

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The observation that type Ia supernovae (SNe Ia) are fainter than expected given their redshifts has led to the conclusion that the expansion of the universe is accelerating. The widely accepted hypothesis is that this acceleration is caused by a cosmological constant or some dark energy field that pervades the universe. We explore what the supernovae data tell us about this hypothesis by answering the question: Can these data be explained with a model in which the strength of gravity varies on a cosmic timescale? We conclude that they can and find that the supernovae data alone are insufficient to distinguish between a model with a cosmological constant and one in which G varies. However, the varying-G models are not viable when other data are taken into account. The topic is an ideal one for undergraduate physics majors.
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I. INTRODUCTION

Physical cosmology is concerned with the large scale structure and evolution of the universe.[1,2] To achieve our current understanding of the universe, the physics of the very small and the physics of the very large are both needed. It is remarkable, for example, that quantum fluctuations that occurred on microscopic scales in the very early universe may have left an imprint on the largest structures in the universe. The observation that the universe contains matter with only trace amounts of antimatter, rather than matter and antimatter in equal amounts, might find its explanation using Earth bound particle accelerators. Dark matter, for which there is much compelling evidence,[3] might yet turn out to comprise weakly interacting particles which may be accessible in laboratories. The relatively recent synergy between the theories of the very small and the very large is a thrilling achievement. However, there is a cloud on the horizon called dark energy.[4,5]

A big surprise came in 1998 when the High-Z Team[6] and the Supernova Cosmology Project[7] independently observed that type Ia supernovae were fainter than expected. After careful consideration of alternative explanations, both teams of researchers interpreted their observations as evidence that the SNe Ia are further away than expected given their redshifts and assuming a decelerating universal expansion. If the SNe Ia are further away than expected, then the average expansion rate of the universe since the Big Bang must be higher than previously thought. Both teams went further: They concluded that the expansion of the universe is accelerating. Today, the broadly accepted hypothesis is that this acceleration is driven by a form of energy called dark energy that pervades the universe. In the simplest model dark energy is identified with the cosmological constant Λ, which appears in the general form of Einstein's theory of gravity, general relativity. In more complicated models,[5] dark energy is modeled as a dynamical field.

Cosmologists have created a compelling and coherent cosmology based on the Friedmann equation,

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{Kc^2}{a^2} + \frac{\Lambda c^2}{3}, \tag{1}$$

and the associated Friedmann–Lemaître–Robertson–Walker (FLRW) metric,[1]

$$ds^2 = c^2dt^2 - a^2(t)\left[\frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right], \tag{2}$$

where $a(t)$ is the dimensionless scale factor, normalized so that $a(t_0)=1$ at the present time $t=t_0$, $\dot{a}\equiv da/dt$, G is the gravitational constant, ρ is the density of all forms of energy[8] excluding the contribution from the cosmological constant Λ, and $-\infty < K < \infty$ is the spatial curvature. The radial coordinate r is defined so that the proper area of a sphere, centered at any conveniently chosen origin, is $A_0 = 4\pi r^2$ at the present time. Symbols with a subscript of zero denote quantities evaluated at $t=t_0$.

The comoving distance χ associated with the radial coordinate r is given by

$$\chi = \int_0^r \frac{dr'}{\sqrt{1-Kr'^2}} = \sin^{-1}(K^{1/2}r)/K^{1/2}, \tag{3}$$

and $d(t)=a(t)\chi$ is the proper distance at time t. By construction, the comoving and proper distances are numerically identical today. The radial coordinate r, comoving distance χ, radius of curvature $K^{-1/2}$, and the proper distance $d(t)$ are conventionally measured in megaparsecs (Mpc). We invert Eq. (3) to obtain

$$r = \sin(K^{1/2}\chi)/K^{1/2}. \tag{4}$$

For a spatially flat universe, that is, one with $K=0$, Eq. (4) simplifies to $r=\chi$.

The standard model of cosmology, with $K=0$ and $\Lambda > 0$, works remarkably well. However, current theory predicts[4] a value of the cosmological constant Λ that exceeds the observed value by a factor of at least 10^{50}. This difficulty motivates the exploration of alternative explanations, such as ones that invoke time-varying "constants."[9] After all, we know of no compelling reasons why the parameters that appear in our current theories of the physical universe should be independent of space and time. From some perspectives, the puzzle is why they should be constant at all.[10]

Another motivation for exploring alternative explanations of the supernovae data is to determine whether they alone are sufficient to distinguish between a model with a cosmological constant and one without, such as the varying-G models we consider in this paper.

A third, rather different motivation, is the pedagogical value of such investigations. This topic is ideally suited for directed study by undergraduate physics majors. It is exciting and lends itself to open-ended exploration. The work we report was undertaken by one of the authors (R.D.), an undergraduate physics major, under the supervision of the other. Ideally, all undergraduates should have the opportunity to engage in authentic research, but many exciting topics require more material than can be mastered in a reasonable amount of time by a busy student. The advantage of cosmology is that it is intrinsically interesting to many students and, provided the topic is chosen carefully and appropriate conceptual approximations are made, interesting cosmological studies can be done using mathematics and concepts that are accessible to a motivated undergraduate student. We fully endorse the idea that for such students, a "mathematics first" approach, followed by applications is less desirable than the "physics first" approach, as advocated by Hartle for general relativity.[2] The cosmological investigation described below was done in that spirit.

This paper explores two simple phenomenological models of varying-G cosmology[9] using the data compiled by Kowalski *et al.*[11] on 307 supernovae. We assume a spatially flat ($K=0$) universe (motivated in part by the expectations from inflation[1]) and set $\Lambda=0$. However, for completeness, we write all expressions in a form that is valid for arbitrary values of K and Λ.

We find fits to the supernovae data that are competitive with the simplest dark energy model. The fact that nondark energy models can account for these data implies that the supernovae data alone are insufficient to establish dark energy as the preferred hypothesis. That hypothesis becomes compelling only when different datasets are analyzed together. Likewise, any varying-G model must fit not only the supernovae data, but must also be in accord with other data. Given our goal to provide an example of a research project that can be conducted in its entirety by an undergraduate student, we restrict the scope to only one other datum: The bounds on $\dot{G}/G$ at our current epoch. We find that our two varying-G models fail the bounds on $\dot{G}/G$, thereby ruling out this form of variation in G. An interesting aspect of the first model is that the scale factor becomes infinite in a finite amount of time. In this model the universe comes to an end in a catastrophic shredding of everything, a doomsday scenario that has been dubbed the *big rip*.[12]

II. SUPERNOVA COSMOLOGY

A key problem in observational cosmology is measuring distances to galaxies. To do so, we need a standard candle and an operational definition of distance. We consider first the standard candle.

A standard candle is a source whose absolute luminosity is known. Type Ia supernovae[13] are currently the best "standard" candles for very large distances. A type Ia supernova is believed to occur when a star in a binary system overflows its Roche lobe (the region within which its matter is gravitationally bound), causing material from it to accrete onto the companion white dwarf. The mass of the white dwarf gradually increases toward the Chandrasekhar limit (of about 1.4 solar masses),[14] triggering runaway nuclear burning within the star that releases more energy in a matter of weeks than the Sun will emit in 10 billion years. In another class of models, an explosion is triggered by the merging of two low-mass white dwarfs. In a third class of models, a carbon-oxygen low-mass white dwarf explodes when the helium, accreted from a companion star, detonates. For a good review of type Ia supernovae models, see Ref. 15. By measuring specific characteristics of the supernovae light curves (plots of brightness as a function of time), it is possible to make empirically derived corrections for the observed variations in SNe Ia brightness and thereby create well-calibrated standard candles.[16]

A. Luminosity distance

The proper distance between two points in space is a well-defined concept, but cannot be measured in practice. Instead, astronomers use a definition of distance based on the flux of energy received on Earth from the luminous object, that is, the energy received per unit area per unit time,

$$f = \frac{L}{4\pi r^2}, \tag{5}$$

where $L=dE/dt$ is the object's luminosity (its rate of total energy emission) and $A_0=4\pi r^2$ is the proper area at $t=t_0$ of the sphere centered at the location once occupied by the supernova. This relation for the flux is valid for a static universe and for a source that emits energy isotropically. In an expanding universe the luminosity L crossing this sphere is diminished by the factor $(1+z)^2$. By definition, the redshift $z \equiv (\lambda_r - \lambda_e)/\lambda_e$, where λ_e and λ_r are the emitted and received wavelengths, respectively. One factor of $1+z$ arises from the reduction in energy of each photon received on Earth relative to the energy it had at emission, yielding $dE \to dE/(1+z)$. The second factor of $1+z$ is due to the reduction in the rate of arrival of photons at Earth, which yields $1/dt \to (1/dt)/(1+z)$. The corrected expression for the flux is

$$f = \frac{L}{4\pi[(1+z)r]^2} \equiv \frac{L}{4\pi d_L^2}, \tag{6}$$

where $d_L \equiv (1+z)r$ is the luminosity distance. For arbitrary values of the curvature K, the radial coordinate r is related to the comoving distance χ via Eq. (4), which reduces to $r=\chi$ when $K=0$.

B. Distance modulus

Astronomers measure energy fluxes. By convention, fluxes are converted into magnitudes m by $f=q10^{-2m/5}=L/(4\pi d_L^2)$, where q is the flux from objects of magnitude zero, with the luminosity distance d_L measured in megaparsecs (Mpc). The absolute magnitude M is defined by $f_M=q10^{-2M/5} = L/(4\pi d_M^2)$, where $d_M=10^{-5}$ Mpc; that is, it is the magnitude of an object viewed from a distance of 10 pc. Astronomers take the logarithm of the ratio $f_M/f=10^{0.4(m-M)} = (d_L/10^{-5})^2$ to arrive at

$$m - M = 5 \log_{10}[d_L/10^{-5}] = 5 \log_{10}[(1+z)r(z)/10^{-5}]. \tag{7}$$

Note that the constant q cancels. The difference $\mu \equiv m-M$ between the apparent magnitude m of a source and its absolute magnitude M is the distance modulus. The analysis of a supernova light curve results ultimately in two measured quantities: μ and z.

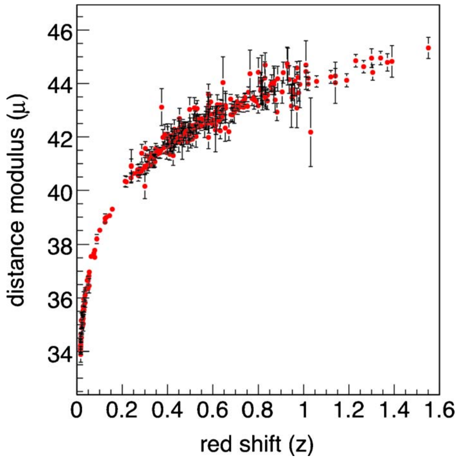

Fig. 1. Measurements of the distance modulus μ and the redshift z for 307 type Ia supernovae (Ref. 11).

The data[11] used in our study are plotted in Fig. 1.[17] The cosmology is contained in the dependence of the radial distance r, or, equivalently, the comoving distance χ, on the redshift z. The redshift is related to the dimensionless scale factor, $a(t)$, as

$$a = 1/(1+z). \tag{8}$$

Given a functional relation between the comoving distance χ and the redshift z, the distance modulus function in Eq. (7) can be fitted to the data in Fig. 1 to extract the parameters of the cosmological model. An equation for the comoving distance χ can be deduced from the FLRW metric in Eq. (2) by noting that light in vacuum travels on null worldlines (for which $ds=0$). Therefore, a light ray from a supernova at redshift z satisfies the relation $cdt=a(t)d\chi$. Hence,

$$\chi(z) = c\int_{(1+z)^{-1}}^{1} \frac{da}{a\dot{a}}, \tag{9}$$

where the light ray was emitted when the scale factor was $a=1/(1+z)$ and received today when it assumes the value unity.

III. A VARYING-G FRIEDMANN EQUATION

Our first assumption is that the universe has zero spatial curvature. Our second assumption is that the Friedmann equation [Eq. (1)] for a $K=\Lambda=0$ universe remains applicable when G is allowed to vary with time; that is, Eq. (1) is a good approximation to some (unknown) exact equation describing the evolution of the scale factor in a universe in which G varies. This assumption is an example of an approximation that renders the problem tractable for an undergraduate student. If we wish to remain strictly within the framework of general relativity, we should be cautious about replacing Eq. (1) by one in which G is a function of time because the Friedmann equation is derived from Einstein's equations,

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}, \tag{10}$$

which do not allow variations[9] in G. The tensors $G_{\mu\nu}$ and $T_{\mu\nu}$ are the components of the Einstein and energy-momentum tensors, respectively, and g is the metric tensor. To allow for a possible variation of G, theories more general than Einstein's are needed, such as scalar-tensor theories[9,19] in which gravity is assumed to couple to a scalar field ϕ. Such theories yield for weak constant coupling the relation $G \propto \phi^{-1}$. This relation yields a modified Friedmann equation with a time-dependent G and additional terms of order $\dot{G}/G$. If the latter terms are sufficiently small, we obtain a Friedmann equation identical in form to the standard one, but with a time-dependent G.

We let $G(t)=G_0 f(a)$, where G_0 is the current value of G and $f(a)$ describes the assumed dependence of G on the scale factor $a(t)$ and therefore cosmic time t. We use the definitions

$$\rho_{c,0} \equiv 3H_0^2/8\pi G_0, \tag{11a}$$

$$\rho_\Lambda \equiv \Lambda c^2/8\pi G_0, \tag{11b}$$

$$\Omega_M(a) \equiv \rho(a)/\rho_{c,0}, \tag{11c}$$

$$\Omega_\Lambda \equiv \rho_\Lambda/\rho_{c,0}, \tag{11d}$$

$$\Omega_0 \equiv \Omega_M(1) + \Omega_\Lambda, \tag{11e}$$

where $\rho_{c,0}$ is the critical density now, $\Omega_M(a)$ is the matter density parameter, and H_0 is the value of the Hubble parameter $H(t) \equiv \dot{a}/a$ today. We may write the modified Friedmann equation as

$$\left(\frac{\dot{a}}{a}\right)^2 = H_0^2[f(a)\Omega_M(a) + (1-\Omega_0)a^{-2} + \Omega_\Lambda], \tag{12}$$

noting that

$$-Kc^2 = H_0^2(1-\Omega_0). \tag{13}$$

With these definitions, we can write the expressions for the comoving distance $\chi(z)$ and the universal time $t(a)$ as

$$\chi(z) = \frac{c}{H_0}\int_{(1+z)^{-1}}^{1} \frac{da}{a^2\sqrt{f(a)\Omega_M(a) + (1-\Omega_0)a^{-2} + \Omega_\Lambda}} \tag{14}$$

and

$$t(a) = \frac{1}{H_0}\int_0^a \frac{dx}{x\sqrt{f(x)\Omega_M(x) + (1-\Omega_0)x^{-2} + \Omega_\Lambda}}. \tag{15}$$

The lifetime of the universe is given by $t_0=t(1)$.

Our third assumption is that the total mass-energy in the universe, whatever its nature, scales in the same way as matter. This assumption implies that $\Omega_\Lambda=0$ and $\Omega_M(a) = \Omega_{M,0}/a^3$, where $\Omega_{M,0}=\Omega_M(1)$ denotes the value of the matter density parameter today. Because we also assume $K=0$, Eqs. (11) and (13) show that $\Omega_0=\Omega_{M,0}=1$. Observations interpreted in the context of the standard cosmology[20] indicate that the matter density parameter $\Omega_{M,0} \approx 0.3$. The difference between $\Omega_0=1$ and $\Omega_{M,0}$ is presumed to be due to the cosmological constant or dark energy. If we wished to be consistent with this value of $\Omega_{M,0}$, while keeping $\Lambda=0$, we need to use a model with $K<0$.

One of our goals is to ascertain whether the SNe data are sufficient to conclude that the $\Lambda>0$ model is preferred. To do so, we need to exhibit another model that works as well.

Here we consider varying-G models with $K=\Lambda=0$ and therefore $\Omega_{M,0}=1$. Alternatively, we could (but do not) consider $\Lambda=0$, $K\neq 0$, models. Note that the curvature term $(1-\Omega_0)a^{-2}$ cannot accelerate the expansion. In a universe dominated by curvature, the Friedmann equation is $\dot{a}=$constant, which implies zero acceleration. To obtain acceleration, we need a term that decreases less rapidly than the curvature term, which is the case for a cosmological constant or for the varying-G models described in the following.

IV. VARYING-G MODELS AND RESULTS

In principle, a model for the variation of G should arise from some deep theory.[9] We proceed in a purely phenomenological manner. Our basic premise is that the supernovae are further away than expected because gravity was weaker in the past and, consequently, the universe decelerated less rapidly than would be the case if G were constant and equal to its current value, G_0.

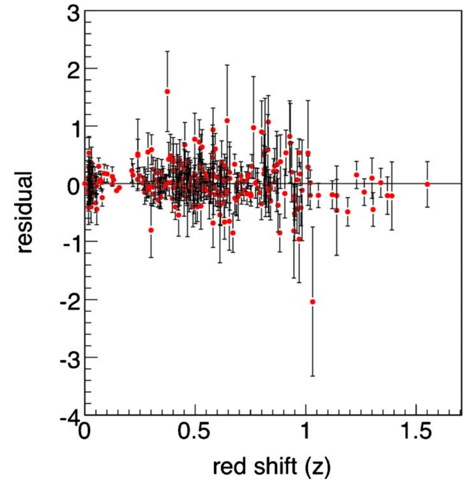

Fig. 2. Residuals, $\mu_n-\mu(z_n)$, for model 1. The residual is the difference between the measured distance modulus μ_n and the fitted function $\mu(z_n)$, evaluated at the measured redshift z_n. The fit has $\chi^2/\mathrm{ND}=313.0/305=1.03$ where ND, the number of degrees of freedom, is 307 data points minus 2 fitted parameters, b and Q.

A. Fits to supernovae data

We studied several forms for the function $f(a)$ in $G(a)=G_0 f(a)$, but we report here results for only two of them, each with a single adjustable, dimensionless, parameter, b. One varying-G model is defined by

$$f(a)=e^{b(a-1)} \quad \text{(model 1)}. \tag{16}$$

In this model, there is no limit to how strong gravity can become. Another model we studied is defined by

$$f(a)=2/(1+e^{-b(a-1)}) \quad \text{(model 2)}, \tag{17}$$

in which G is limited to twice its current value in the distant future. We normalized both models so that $G(a)$ assumes its current value, G_0, when $a=1$. For $K=0$ models, the distance modulus [Eq. (7)] may be written as

$$\mu(z,b,Q)=5\log_{10}[(1+z)H_0 r(z)/c]+Q, \tag{18}$$

where the offset Q determines the vertical location of the modulus curve.[21] Note that $H_0 r(z)/c$ is dimensionless and is independent of the Hubble constant.

We evaluate Eqs. (14) and (15) for model 1 with $K=0$ (that is, $\Omega_0=1$) and $\Omega_\Lambda=0$. We find

$$\chi(z)=r(z)=\frac{c}{H_0}e^{b/2}\sqrt{2\pi/b}\,[\mathrm{erf}(\sqrt{b/2}) - \mathrm{erf}(\sqrt{b(1+z)^{-1}/2})] \tag{19}$$

and

$$t(a)=\frac{1}{H_0}[e^{b/2}(\sqrt{2\pi/b}\,\mathrm{erf}(\sqrt{ab/2})-2\sqrt{a}e^{-ab/2})]/b. \tag{20}$$

This model exhibits a striking feature: The scale factor becomes infinite in a finite amount of time. For model 2, the integrals in Eqs. (14) and (15) are evaluated numerically using the midpoint rule.[22]

We fit Eq. (18) to the SNe data in Fig. 1 by minimizing the function

$$\chi^2=\sum_{n=1}^{307}[\mu_n-\mu(z_n,b,Q)]^2/\sigma_n^2, \tag{21}$$

with respect to the parameters b and Q, where $1\leq n\leq 307$ labels the nth supernova at redshift z_n and distance modulus μ_n, measured with an uncertainty of $\pm\sigma_n$. The minimization of Eq. (21) is done using the program TMINUIT.[23]

For model 1, we obtain the result shown in Fig. 2. The fit gives the value $b=2.09\pm 0.08$, from which we infer a lifetime of $t_0=15.1\pm 0.3$ $(70\ \mathrm{km\ s^{-1}\ Mpc^{-1}}/H_0)$ Gyr.[24] The fact that the χ^2 per number of degrees (ND) of freedom (χ^2/ND) is 1.03 suggests that the modulus uncertainties are estimated correctly and that model 1 provides an excellent description of the data.[25] A similarly good fit is found for model 2, which yields residuals almost identical to those shown in Fig. 2 for model 1. This fit yields $b=3.27\pm 0.11$, with $\chi^2/\mathrm{ND}=316/305=1.04$. We find $t_0=16.2\pm 0.4$ $(70\ \mathrm{km\ s^{-1}\ Mpc^{-1}}/H_0)$ Gyr. For the simplest dark energy model, for which $f(a)=1$ and $\Omega_M(a)=(1-\Omega_\Lambda)/a^3$ with $\Omega_\Lambda>0$, we find $\Omega_\Lambda=0.71\pm .02$ and $t_0=14.0\pm 0.3$ $(70\ \mathrm{km\ s^{-1}\ Mpc^{-1}}/H_0)$ Gyr, consistent with the accepted results.[4] The χ^2 per number of degrees of freedom of the fit is $310/305=1.02$.

Because there is no compelling statistical basis to reject any of these models, we conclude that the supernovae data alone are insufficient to distinguish between them. However, these data when analyzed along with other data[4] are consistent with a simple cosmology in which dark energy mimics a cosmological constant with $\Omega_\Lambda\approx 0.7$. The varying-G models should likewise be analyzed along with other data to determine if a consistent picture emerges. The fact that a $K=\Lambda=0$ model requires $\Omega_{M,0}=1$, while the preferred value from galaxy and galaxy cluster measurements is $\Omega_{M,0}=0.3$ is an indication of a problem.

A systematic analysis of the relevant data is beyond the scope of this paper. Instead, we illustrate the importance of including other data by comparing the predicted fractional variation of $\dot{G}/G$ at the present epoch, with the available bounds.

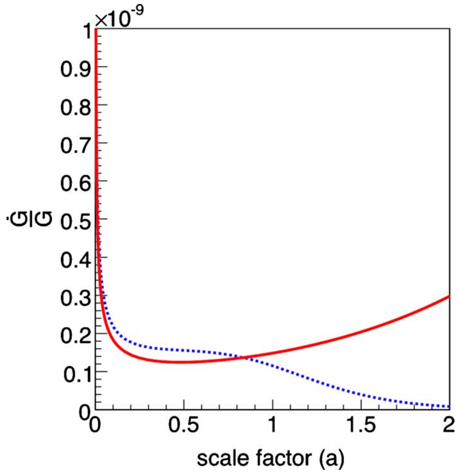

Fig. 3. Fractional change in G per year for model 1 (solid curve) and model 2 (dashed curve) as a function of the dimensionless scale factor $a(t)$.

B. Bounds on the variation of *G*

The possible variation of G is usually characterized by the quantity $\dot{G}/G$, which, in terms of the logarithmic derivative of the function $f(a)$, is given by

$$\frac{\dot{G}}{G} = \frac{1}{G}\frac{dG}{da}\dot{a} = H_0\frac{d\ln f}{da}, \tag{22}$$

where we have used the fact that $H_0 = \dot{a}/a = \dot{a}$ at the present epoch. Figure 3 shows $\dot{G}/G$ as a function of the scale factor for models 1 and 2. We see that at $a=1$, $\dot{G}/G$ is equal to 1.5×10^{-10} and 1.15×10^{-10} y^{-1}, respectively. These values for $\dot{G}/G$ are one to three orders of magnitude larger than the upper bounds, which range from about $10^{-10}-10^{-13}$ y^{-1}, depending on the method used to extract the bound.[26]

V. DISCUSSION

We have presented an investigation of varying-G cosmological models that serve as examples of interesting research problems that are well matched to the sophistication of an undergraduate.

The two phenomenological models, in which the strength of gravity increases with cosmic time, provide excellent fits to the type Ia supernovae data. We therefore conclude that the supernovae data alone cannot establish the dark energy hypothesis unambiguously. However, both our varying-G models fail to satisfy the bounds on $\dot{G}/G$. Consequently, the particular variation of G described by these models is ruled out. We can make an even stronger statement: All varying-G models that give rise to accelerated expansion and that are based on the FLRW metric and the Friedmann equation are ruled out by these bounds.[27] Consider, for example, matter-dominated models, for which the Friedmann equation is $H^2 \sim G/a^3 \rightarrow G \sim a\dot{a}^2$. This equation yields $\dot{G}/G \sim H + 2\ddot{a}/\dot{a}$, from which we conclude that $\dot{G}/G \gtrsim H$. But a value of $\dot{G}/G$ of the order of H_0 is inconsistent with the bounds on $\dot{G}/G$, which are less than H_0 by one to three orders of magnitude.

The inability to distinguish between models is inherent in the Friedmann equation because the latter is sensitive only to the total energy density of the universe and is agnostic with respect to how the energy density arises. This independence can be seen by writing Eq. (1) as

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}(\rho_M + \rho_K + \rho_\Lambda), \tag{23}$$

where ρ_M, ρ_K, and ρ_Λ are the density contributions from matter, the curvature, and the cosmological constant, respectively. The Hubble parameter $\dot{a}/a$ is related to the sum of $\rho_M+\rho_K+\rho_\Lambda$, not the individual components, or, equivalently, to the sum

$$\Omega \equiv f(a)\Omega_M(a) + (1-\Omega_0)a^{-2} + \Omega_\Lambda. \tag{24}$$

Therefore, it is possible to entertain different interpretations of the total energy density. For example, any model based on the Friedmann equation can be interpreted as one in which matter, perhaps of several different sorts, is either created, destroyed, or both as the universe evolves. Consider, for example, the simple cosmological constant dark energy model, for which $f(a)=1$, $\Omega_0=1$, and the total energy density is given by $\Omega=(1-\Omega_\Lambda)/a^3+\Omega_\Lambda$. This relation can be rewritten as $\Omega=\Omega'_M(a)/a^3$ with $\Omega'_M(a)=1-\Omega_\Lambda+\Omega_\Lambda a^3$. Because a^{-3} is the dilution factor for matter, the function $\Omega'_M(a)$ describes an increasing matter density in a comoving volume, which can be interpreted as the creation of matter as the universe expands. Alternatively, as done here, we can maintain the mass continuity equation, in which case matter is neither created nor destroyed and $\Omega=\Omega_{M,0}/a^3$, but allow G to vary like $f(a)\sim 1+a^3(1-\Omega_{M,0})/\Omega_{M,0}$. Because of the invariance of the Friedmann equation with respect to such changes in interpretation, it is necessary to impose constraints on the cosmological parameters to remove the model degeneracy. Such constraints can come from other data, other equations, or both.

It might seem odd that the strengthening of gravity with time leads not to the eventual gravitational collapse of the universe, but rather to its accelerating expansion. The reason is that every form of energy contributes to the geometry of spacetime. A model in which the strength of gravity changes with time is equivalent to another model in which the energy density changes in a specific way. If the energy density dilutes more rapidly than a^{-2}, then the expansion will slow down. If the strength of gravity increases such that in the equivalent constant-G model, the energy dilutes more slowly than a^{-2}, the expansion will accelerate. In our varying-G models, the effective energy density increases with time.

For model 1, the increasing strength of gravity leads to a startling prediction: A catastrophic end to such a universe. This conclusion follows from the limit $a\rightarrow\infty$ of the lifetime expression [Eq. (20)]. We find that

$$t(a\rightarrow\infty) = \frac{1}{H_0}\exp(b/2)\sqrt{2\pi/b}/b. \tag{25}$$

According to this model, the universe has a finite lifetime of about 33 Gyr and will tear itself to pieces in its final moments. Such behavior is a feature of cosmological models containing phantom energy.[12] Within regions that are dominated by nongravitational forces, the effect of a cosmological constant does not change with time and, consequently, the accelerating universal expansion will not disrupt already bound systems. In contrast, as the universe ages, the effect of phantom energy increases in any finite volume of space.

Eventually, this increasing phantom energy precipitates an escalating cascade of destruction at ever smaller scales until everything is torn asunder. We can only hope that phantom energy is just that.

ACKNOWLEDGMENTS

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[17] We have described the bolometric magnitude, that is, the magnitude of a star assuming we can measure the flux across all wavelengths. In practice, fluxes are measured in wavelength bands defined by standard filters, such as the B-band filters (Ref. 18). An observed supernova spectrum is redshifted with respect to the spectrum in the rest frame of the supernova. Therefore, a filter will transmit a flux that differs from the one that would be measured in the supernova's rest frame. Astronomers use K corrections to map the measured flux back to its value in the object's rest frame. Given a model of the object's spectrum, it is possible, in principle, to infer the bolometric flux and hence the bolometric magnitude of the object. The distance modulus data compiled in Ref. 11 are derived from B-band magnitude data.

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[21] The absolute luminosity of a supernova cannot be determined independently of the Hubble constant H_0. Consequently, in the fit of the modulus function to the data, it is only the shape of the function that contains useful information about the cosmology. The offset Q depends both on H_0 as well as on the flux corrections.

[22] If the interval $[a,b]$ is divided into N intervals of width $h=(b-a)/N$, the midpoint rule is $\int_a^b f(x)dx \approx h\Sigma_{i=1}^N f(a+(i-0.5)h)$.

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[24] The lifetimes can be computed given values for the parameters b and H_0. However, because we cannot extract a value of H_0 from the fits, we compute the lifetimes using the nominal value 70 km s^{-1} Mpc^{-1} for the Hubble constant. We write all lifetimes in terms of the parameter H_0 to make clear how the numerical values will change if H_0 differs from the nominal value.

[25] If the reported modulus uncertainties are Gaussian distributed, we expect χ^2 to be sampled from a probability density with mean $N-P\equiv$ ND, where $N=307$ is the number of data points and $P\approx 2$ is the number of adjustable parameters. ND is the number of degrees of freedom. Therefore, for a fit that neither overfits nor underfits, we expect $\langle\chi^2\rangle/\mathrm{ND}\approx 1$. The quantity P would be exactly equal to 2 if the constraints that define the parameter estimates were linear in the parameters.

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