1st set of orrections to Relativity, Gravitation and Cosmology 2e
by Ta-Pei Cheng (June 2010) — ten pages

[A] Inserting missing square brackets: (8 pages)

- p.36, right hand side of Eq (2.48): Insert a missing square bracket so that the displayed equation reads as

$$\partial'_\mu = \sum_\nu [\bar{\mathbf{L}}]_\mu{}^\nu \partial_\nu, \tag{2.48}$$

- p.37, just below Eq (2.49): Insert a missing square bracket so that the inline equation reads as " $[\bar{\mathbf{L}}] = [\mathbf{L}]^{-1}$."

- p.45, right hand side of Eq (3.17) and the line below: Insert 2 missing square brackets so that the displayed equation and the line below read as

$$x'^\mu = [\mathbf{L}]^\mu{}_\nu x^\nu \tag{3.17}$$

[**L**] denotes the 4×4 Lorentz transformation matrix,..."

- p.46, right hand side of Eq (3.18): Insert a missing square bracket so that the displayed equation reads as

$$A'^\mu = [\mathbf{L}]^\mu{}_\nu A^\nu. \tag{3.18}$$

- p.47, Eq (3.27) and the line below as well as Eq (3.29): Insert 3 missing square brackets so that the displayed equations read as

$$\frac{dx'^\mu}{dt'} \neq [\mathbf{L}]^\mu{}_\nu \frac{dx^\nu}{dt} \quad \text{as } t' \neq t. \tag{3.27}$$

While dx^μ is a 4-vector ($dx'^\mu = [\mathbf{L}]^\mu{}_\nu dx^\nu$), the ordinary time......

....

$$U'^\mu = [\mathbf{L}]^\mu{}_\nu U^\nu. \tag{3.29}$$

- p.59 , Eq (3.57) in Problem 3.5: Insert 3 missing square brackets in the first line so that the displayed equation reads as

$$\begin{aligned} [\mathbf{R}(\theta_1)][\mathbf{R}(\theta_2)] &= [\mathbf{R}(\theta_1+\theta_2)] \\ [\mathbf{L}(\psi_1)][\mathbf{L}(\psi_2)] &= [\mathbf{L}(\psi_1+\psi_2)]. \end{aligned} \tag{3.57}$$

- p.75, Eq (4.41) and Eq (4.42): Insert 3 missing square brackets so that the displayed equations read as

$$(d\phi) \backsimeq \frac{(c_1 - c_2)dt}{dy} \simeq \frac{d[c(r)](dx/c)}{dy}. \tag{4.41}$$

  Working in the limit of weak gravity with small $\Phi(r)/c^2$ (or equivalently $n \simeq 1$), we can relate $d[c(r)]$ to a change of index of refraction as

$$d[c(r)] = cd[n^{-1}] = -cn^{-2}dn \simeq -cdn. \tag{4.42}$$

- p.87, 3rd line below Eq (5.27): Insert a missing square bracket so that the inline mathematical expression reads as $[(\partial L/\partial \dot{x})\delta x]_{\lambda_i}^{\lambda_f}$.

- p.95-97, Eq.(48) to Eq (5.55): Insert 8 missing square brackets in

LHS	of	Eq.
$[ds^2]^{(k)}_{2\mathrm{D},\chi}$		(5.48)
$[ds^2]^{(k)}_{2\mathrm{D},\chi}$		(5.50)
$[ds^2]^{(k)}_{2\mathrm{D},\xi}$		(5.51)
$[ds^2]^{(k=0)}_{2\mathrm{D}}$		(5.52)
$[ds^2]^{(k=0)}_{3\mathrm{D}}$		(5.52)
$[ds^2]^{(k)}_{3\mathrm{D},\chi}$		(5.53)
$[ds^2]^{(k)}_{3\mathrm{D},\chi}$		(5.54)
$[ds^2]^{(k)}_{3\mathrm{D},\xi}$		(5.55)

- p.116, Problem 6.8: Insert 3 missing square brackets in the expression "energylengthmass^{-2}" so that it reads as "[energy][length][mass]$^{-2}$ "

- p.123, the sentence between Eq (7.24) and Eq (7.25): Insert 2 missing square brackets so that the two lines read as

  "A comparison of 7.24) with (7.23) leads to $dz = \pm\left[r^*/\left(r - r^*\right)\right]^{1/2} dr$, which can be integrated to yield $z = \pm 2\left[r^*\left(r - r^*\right)\right]^{1/2}$, or"

- p.207, Eq.(10.6): Insert a missing square bracket so that the displayed equation reads as

$$\rho_{\mathrm{c}}(t) = \frac{3}{8\pi G_{\mathrm{N}}}\frac{\dot{a}^2}{a^2} = \frac{3[H(t)]^2}{8\pi G_{\mathrm{N}}} \tag{10.6}$$

- p.212, right hand sides of Eq (10.23) and Eq (10.24): Insert 3 missing square brackets so that the displayed equations read as

$$\rho(t) = \rho_0[a(t)]^{-3(1+w)}. \tag{10.23}$$

and

$$\rho_{\rm M}(t) = \rho_{\rm M,0}[a(t)]^{-3} \quad \text{and} \quad \rho_{\rm R}(t) = \rho_{\rm R,0}[a(t)]^{-4}. \tag{10.24}$$

- p.227, Eq.(10.68): Insert a missing square bracket so that the displayed equation reads as

$$1 = \frac{\rho_{\rm R}}{\rho_{\rm M}} = \frac{\rho_{\rm R,0}}{\rho_{\rm M,0}}[a(t_{\rm RM})]^{-1} = \frac{\Omega_{\rm R,0}}{\Omega_{\rm M,0}}(1+z_{\rm RM}) \simeq \frac{1+z_{\rm RM}}{1.1\times 10^4}. \tag{10.68}$$

- p.229, the line below Eq.(10.76): Insert a missing square bracket in the inline mathematical expression so that the line reads as: "because the neutrino effective spin degrees of freedom $g_\nu^* = \frac{7}{8}\,[3\times(1+1)]$ ..."

- p.244, Eq.(11.15): Insert 2 missing square brackets so that the displayed equation reads as

$$[1-\Omega(t)] = \frac{-kc^2}{[\dot{a}(t)]^2 R_0^2}. \tag{11.15}$$

- p.255, first line in Eq.(11.31): Insert the missing square bracket in the denominator so that the line reads as:

$$\alpha_1 \approx \frac{\lambda_1}{d(t_\gamma)} = \frac{c_{\rm s}(1+z_\gamma)^{-1/2}}{c[(1+z_0)^{-1/2}-(1+z_\gamma)^{-1/2}]}$$

- p.260, Eq.(11.39): Insert 2 missing square brackets so that the displayed equation reads as

$$\begin{aligned} H(z) &= H_0[\Omega_{\rm R,0}(1+z)^4 + \Omega_{\rm M,0}(1+z)^3 + \Omega_\Lambda + (1-\Omega_0)(1+z)^2]^{1/2} \\ &\simeq H_0[\Omega_{\rm M,0}(1+z)^3 + \Omega_\Lambda + (1-\Omega_{\rm M,0}-\Omega_\Lambda)(1+z)^2]^{1/2}. \end{aligned} \tag{11.39}$$

- p.260, Eq.(11.42): Insert a missing square bracket so that the displayed equation reads as

$$t_0 = t_{\rm H}\int_0^1 \frac{da}{[\Omega_{\rm R,0}a^{-2} + \Omega_{\rm M,0}a^{-1} + \Omega_\Lambda a^2 + (1-\Omega_0)]^{1/2}} \tag{11.42}$$

- p.283, sidenote 5: Insert 4 missing square brackets so that the side note reads as

  "[5]We remind ourselves that mathematical objects with indices, being the components of tensors and matrices, are ordinary numbers; they are communitive for example, $A_\nu \, [\mathbf{L}]^{\nu}{}_{\mu} = [\mathbf{L}]^{\nu}{}_{\mu} \, A_\nu$, even though their corresponding matrices are not generally be communitive, e.g., $A \, [\mathbf{L}] \neq [\mathbf{L}] \, A$. This means that we can move them (the components) around with ease."

- p.283, sidenote 7: Insert 4 missing square brackets so that the side note reads as

  "[7]For the case of the inverse metric $g^{\mu\nu}$, we would be working with $[\mathbf{L}]$ instead of $[\mathbf{L}^{-1}]$. Also, all the matrices under discussion, $[\mathbf{L}^{-1}]$ and $[\mathbf{g}]$, are square matrices."

- p.283-284, the two paragraphs following Eq.(12.18): Insert a total 22 square brackets so that these two paragraphs and Eq.(12.19) read as:

  "The matrix $[\mathbf{L}^{-1}]^\intercal$ is the transpose of $[\mathbf{L}^{-1}]$; this comes about because in (12.17) we need to interchange the row and column (*i.e.,* the first and second) indices α and μ of the first $[\mathbf{L}^{-1}]$ matrix in order to have the proper summation of the products of column/row elements in a matrix multiplication.

  In a flat space, such as the Minkowski space of SR, we can always use a coordinate system such that the metric is position independent, $[\mathbf{g}] = [\boldsymbol{\eta}]$, as in (3.15). We can also require the coordinate transformation that leaves the metric invariant. Keeping the same metric $\eta_{\mu\nu}$ means we require coordinate transformations not to change the geometry, not to take us out of Minkowski spacetime. This is consistent with our conception of spacetime in special relativity as being a fixed stage on which physical processes taking place without effecting the spacetime background. With $[\mathbf{g}'] = [\mathbf{g}] = [\boldsymbol{\eta}]$, Eq. (12.18) becomes

  $$[\boldsymbol{\eta}] = [\mathbf{L}^{-1}]^\intercal \, [\boldsymbol{\eta}] \, [\mathbf{L}^{-1}] = [\mathbf{L}^{-1}] \, [\boldsymbol{\eta}] \, [\mathbf{L}^{-1}]^\intercal. \qquad (12.19)$$

  The relation can be regarded as the generalized orthogonality condition as it reduces to the familiar orthogonality property of $[\mathbf{L}^{-1}]^\intercal [\mathbf{L}^{-1}] = [\mathbf{L}^{-1}][\mathbf{L}^{-1}]^\intercal = [\mathbf{1}]$ for the Euclidean space with $[\mathbf{g}]$ replaced by $[\mathbf{1}]$. (In Problem 12.8, we have used such conditions to derive the explicit form of the Lorentz transformation.)"

- p.301, the paragraphs containing Eq.(13.14): Insert a total 14 square brackets so that the paragraph reads as:

  "In summary we are interested in general coordinate transformation $x'^{\mu} = x'^{\mu}(x)$ that leaves the infinitesimal length ds^2 of (13.1) invariant. This is sometimes described as a general reparametrization. Under such transformation $[\mathbf{L}] = [\partial x'/\partial x]$ the metric tensor transforms, as shown in (13.12), as

  $$g'_{\mu\nu} = g_{\alpha\beta}\left[\mathbf{L}^{-1}\right]^{\alpha}{}_{\mu}\left[\mathbf{L}^{-1}\right]^{\beta}{}_{\nu}, \tag{13.14}$$

  or in matrix language $[\mathbf{g}'] = [\mathbf{L}^{-1}]^{\intercal}[\mathbf{g}][\mathbf{L}^{-1}]$. We are interested the general reparametrizations that are "smooth" so that operations as $\partial x'/\partial x$ exists. Also, in the small and empty space, we should still have the residual transformation that leaves the metric invariant: $[\mathbf{g}'] = [\mathbf{g}] = [\boldsymbol{\eta}]$. That is, the metric remains to be Minkowskian. Recall from Chapters 12 and 3, the condition $[\boldsymbol{\eta}] = \left[\mathbf{L}_0^{-1}\right]^{\intercal}[\boldsymbol{\eta}]\left[\mathbf{L}_0^{-1}\right]$ informs us that $[\mathbf{L}_0]$ must be the Lorentz transformation. That is, in a local small empty space the transformation must be reducible to a Lorentz transformation. In this sense the general coordinate transformation we will study can be regarded as "local Lorentz transformation" — an independent Lorentz transformation at every spacetime point.

- p.321, Eq (14.8): Insert 2 missing square brackets so that the displayed equation read as

  $$[\nabla^2\Phi = 4\pi G_{\mathrm{N}}\rho] \rightarrow [?],. \tag{14.8}$$

- p.321-322, Eq.(14.11) and the paragraph below: Insert a total 9 square brackets, as well as make the letters O, g, T bold faced $\mathbf{O}, \mathbf{g}, \mathbf{T}$, so that the equation and the paragraph read as:

  "

  $$[\hat{\mathbf{O}}\mathbf{g}] = \kappa[\mathbf{T}]. \tag{14.11}$$

  Namely, some differential operator $[\hat{\mathbf{O}}]$ acting on the metric $[\mathbf{g}]$ to yield the energy–momentum tensor $[\mathbf{T}]$ with κ being the "conversion factor" proportional to Newton's constant G_{N} that allows us to relate energy density and the spacetime curvature. Since we expect $[\hat{\mathbf{O}}\mathbf{g}]$ to have the Newtonian limit of $\nabla^2\Phi$, the operator $[\hat{\mathbf{O}}]$ must be a second derivative operator. Besides the $\partial^2 g$ terms, we also expect it to contain nonlinear terms of the type of $(\partial g)^2$. The presence of the nonlinear $(\partial g)^2$ terms is suggested by the fact that energy, just like mass, is a source of gravitational fields, and gravitational fields themselves hold energy—just as electromagnetic fields hold energy, with density being quadratic in fields $(\vec{E}^2+\vec{B}^2)$. That is, gravitational field energy density must be quadratic in the gravitational field strength, $(\partial g)^2$. In terms of Christoffel symbols $\Gamma \sim \partial g$, we anticipate $[\hat{\mathbf{O}}\mathbf{g}]$ to contain not only $\partial\Gamma$ but also Γ^2 terms as well. Furthermore, because the right-hand side (RHS) is a symmetric tensor of rank 2 which is covariantly constant, $D_\mu T^{\mu\nu} = 0$ (reflecting energy–momentum conservation), $[\hat{\mathbf{O}}\mathbf{g}]$ on the LHS must have these properties also. The basic properties that the LHS of the field equation must have, in order to match those of $T^{\mu\nu}$ on the RHS, are summarized below:"

- p.344, the 2nd paragraph and 3rd line: insert a missing square bracket so that the inline equation reads as: "...separation as $ds = [1\pm\frac{1}{2}h_\times \sin\omega(t-z/c)]\xi$. The generalization to ...."

- p.349, the 3rd line above Eq.(15.46): insert a missing square bracket so that the inline equation reads as: "...of $\left\langle \tilde{h}_+\partial_0\tilde{h}_+\right\rangle \propto \langle \sin\left[2\omega\left(t-z/c\right)\right]\rangle = 0$. Hence we will drop the $\tilde{h}_+\partial_0\tilde{h}_+$ terms ..."

- p.357, Eq.(15.76): insert a missing square bracket on the RHS of the equation so that it reads as "

$$\tilde{I}^{\rm TT}_{ij}\tilde{I}^{\rm TT}_{ij} = \frac{1}{2}\left[2\tilde{I}_{ij}\tilde{I}_{ij} - 4\tilde{I}_{ik}\tilde{I}_{il}n_kn_l + \tilde{I}_{ij}\tilde{I}_{kl}n_in_jn_kn_l\right]. \qquad (15.76)$$

- p.362, Review Question 4 in Chapter 3: insert a missing square bracket so that the inline equation reads: "... transforms as $A^\mu \rightarrow A'^\mu = [\mathbf{L}]^\mu_{\ \nu} A^\nu$. See discussion ..."

- p.367, Review Question 10 in Chapter 9: insert a missing square bracket on LHS and a parenthesis on RHS so that the inline equation reads as: "... leads to $[a(t_0)/a(t_{\rm em})] = (1+z)$."

- p.369, Review Questions 3 and 4 in Chapter 12: insert 4 missing square brackets so that they read as:

3. $A'^{\mu} = [\mathbf{L}]^{\mu}{}_{\nu} A^{\nu}$ and $A'_{\mu} = [\mathbf{L}^{-1}]_{\mu}{}^{\nu} A_{\nu}$.

4. $T'^{\mu}_{\ \nu} = [\mathbf{L}]^{\mu}{}_{\lambda} [\mathbf{L}^{-1}]_{\nu}{}^{\rho} T^{\lambda}{}_{\rho}$

- p.372, Problem 2.3 part (b): insert 11 missing square brackets so that the solution reads as:

  (b) $[\bar{\mathbf{L}}]$ can be found by substituting into $\delta^{\nu}_{\mu} = \partial(x'_{\nu})/\partial x'_{\mu} \equiv \partial'_{\mu} x'^{\nu}$ the respective Lorentz transformations $[\mathbf{L}]$ and $[\bar{\mathbf{L}}]$ for coordinates and coordinate derivatives Eqs (2.47) and (2.48):

$$\begin{aligned} \delta^{\nu}_{\mu} &= \partial'_{\mu} x'^{\nu} = \sum_{\lambda,\rho} \left([\bar{\mathbf{L}}]_{\mu}{}^{\lambda} \partial_{\lambda}\right) \left([\mathbf{L}]^{\nu}{}_{\rho} x^{\rho}\right) \\ &= \sum_{\lambda,\rho} [\bar{\mathbf{L}}]_{\mu}{}^{\lambda} [\mathbf{L}]^{\nu}{}_{\rho} \delta^{\rho}_{\lambda} = \sum_{\lambda} [\bar{\mathbf{L}}]_{\mu}{}^{\lambda} [\mathbf{L}]^{\nu}{}_{\lambda}. \end{aligned} \tag{2}$$

  Namely, $\mathbf{1} = [\bar{\mathbf{L}}][\mathbf{L}]$. Thus, the transformation for the coordinate derivative operators is just the inverse shown in (1).

- p.394, the last displayed equation in Problem 10.5: insert a missing square bracket so that the displayed equation reads as:

$$m - M = 5 \log_{10} \frac{2cH_0^{-1}(1 + z - [1+z]^{1/2})}{10 \text{ pc}}.$$

- p.395, the top paragraph following Eq.(26) and part (a) of Problem 10.8: Insert a total 4 square brackets so as to read as:

  "which, for $t_0 = 2/(3H_0)$, agrees with the result obtained in Problem 10.5. For a radiation-dominated flat universe $x = \frac{1}{2}$ we have $d_{\mathrm{p}}(t_0) = 2ct_0[1 - (1+z)^{-1}]$. NB: These simple relations between redshift and time hold only for a universe with a single-component on energy content; moreover, it does not apply to the situation when the equation-of-state parameter is negative ($w = -1$), even though the energy content is a single-component case.

10.8 Scaling behavior of number density and Hubble's constant

(a) For material particles the number density scales as the inverse volume factor,.$[n(t)/n_0] = [a(t)]^{-3}$. The basic relation (9.50) between scale factor and redshift leads to $[n(t)/n_0] = (1+z)^3$. This scaling property also holds for radiation because $n \sim T^3 \sim a^{-3}$ as given in (10.35)."

- p.403, last line in Problem 13.7, insert a missing square bracket so that the inline equation on the last line reads: $[D_\alpha, D_\beta]\, A_\mu = -R^\lambda{}_{\mu\alpha\beta} A_\lambda$

- p.403, Problem 13.9, insert 8 missing square bracket so that the first paragraph reads as: "Write the curvature tensor as $R_{\{[\mu\nu],[\alpha\beta]\}}$ to remind ourselves the symmetry properties of (13.69) to (13.71): antisymmetry of Eq (13.69) as $[\mu\nu]$, that of (13.70) as $[\alpha\beta]$, and the symmetry of (13.71) as $\{[\mu\nu], [\alpha\beta]\}$. An $n \times n$ matrix has $\frac{1}{2}n\,(n+1)$ independent elements if it is symmetric, and $\frac{1}{2}n\,(n-1)$ elements if antisymmetric. Hence, for the purpose of counting independent components, we can regard $R_{\{[\mu\nu],[\alpha\beta]\}}$ as a $\frac{1}{2}n\,(n-1)$ by $\frac{1}{2}n\,(n-1)$ matrix, which is symmetric...."

- p.406, 2nd line in part (2) of Problem 13.12: insert the missing *double square bracket* so that the inline symbol reads as $[D_\lambda, [D_\mu, D_\nu]]$.

[B] Other corrections: (2 pages)

- p.70, the line above Eq (4.20): Change (3.48) to (3.47).
- p.88, Eq.(5.30): Change "sigma σ" to "lambda λ" to read:

$$\frac{d}{d\lambda}\frac{\partial L}{\partial \dot{x}^{a}} - \frac{\partial L}{\partial x^{a}} = 0.$$

- p.96, the line above Eq (5.54) after the word "or,": Insert a clause " in more compact form,"
- p.108, at the end of the paragraph just above Box 6.2: Insert the sentence of "We note that $\Phi/c^2 = G_{\rm N}M/rc^2$ is just the ε parameter introduced in Eq (4.18)."
- p.124, the first sentence below the subtitle **7.2.1 Light ray deflection: GR vs. EP:** Insert the missing letter "t" in the word "light-like".
- p.131, Eq.(7.46): Insert a missing minus sign in front of the middle term $g_{00}c\dot{t}$ so that the equation reads as

$$\kappa \equiv -g_{\mu\nu}\dot{x}^{\mu}K^{\nu}_{(t)} = -g_{00}c\dot{t} = \left(1 - \frac{r^*}{r}\right)c\dot{t}, \qquad (7.46)$$

- p.131, immediately following Eq.(7.46), before the expression "and, for $\theta = \pi/2$, : " Insert the clause: "which is particle's energy E/mc, " so that this line reads as "which is particle's energy E/mc, and, for $\theta = \pi/2$,"
- p.131, immediately after Eq.(7.46): Insert the clause "which is the orbital angular momentum."
- p.133, immediately after Eq.(7.59): Correct the expression for α so it reads as "$\alpha = l^2/\left(G_{\rm N}Mm^2\right)$".
- p.156, 3rd line below Eq.(8.35): Replace the two words "centrifugal barrier" by "rotational kinetic energy" so that the line reads as "second term the rotational kinetic energy, the last term is a new GR contribution."

- p.198, 2nd line above Eq.(9.39): Insert a square bracket containing a qualifying clause so that the line reads as: "ξ and cosmic time t [*i.e.*, the separation between emitter and receiver on the spacetime surface of a fixed time] can be calculated from...."

- p.279, 4th line from the bottom of the page: Correct the word "altered" to "unaltered" so that the line reads "relational form of the equation is unaltered under such transformations."

- p.363, at the end of Review Question 2 in Chapter 5: Correct the equation number from Eq.(4.16) to Eq.(5.16).

- p.375, 3rd line in Problem 3.6: Correct the equation number from (3.58) to (3.57).

- p.377, 2nd line from the bottom of the page, in Problem 4.3 part (a): Insert the missing superscript 2 over the symbol r_s so that the inline equation reads as "$v_s^2/r_s = G_\mathrm{N}M_\oplus/r_s^2$."

- p.385, the second displayed equation from the top, in Problem 7.3: Replace the symbol λ^2 by j^2 in the fraction, with r^2 being the denominator, so that the equation reads

$$\left(\frac{dr}{d\lambda}\right)^2 + \left(1 - \frac{r^*}{r}\right)\frac{j^2}{r^2} = \kappa^2.$$

- p.386, Problem 7.7 (a): Replace in 4 separate instances the Greek letter λ by the Latin letter j so that the four lines, together with the first part of the displayed equation, read as

  (a) The orbit equation (7.59) for the variable $u \equiv 1/r$ has $u' = 0$ corresponding to a circular orbit case: $u^2 - (r^*c^2/j^2)\,u - r^*u^3 =$ constant, with $j = l/m = r^2 d\phi/d\tau$. If we differentiate this equation, we have $(r^*c^2/j^2) = 2u - 3r^*u^2$. Putting in the specific value $u = 1/R$, it implies

$$R^4\left(\frac{d\phi}{d\tau}\right)^2 = j^2 = \frac{r^*c^2R}{2}\left(1 - \frac{3r^*}{2R}\right)^{-1}$$