Corrections to Relativity, Gravitation and Cosmology 2e
Cheng (10-2015)

- p.29, 2nd line after the subsection heading **Inverse Lorentz transformation**: Interchange the two expressions (x, t) and (x', t') so the first sentence will read as

  "In Eqs. (2.34) and (2.36) the coordinates (x', t') are expressed in terms of (x, t)."

- p.33, Sidenote 30: Insert a bracket on the 2nd line the right hand side of displayed equation so it reads as

$$\begin{aligned} \Delta t' &= \gamma \left(\Delta t - vL/c^2 \right) \\ &= (\gamma L/v) \left[\left(1 - \gamma^{-1} \right) - v^2/c^2 \right] \\ &= -(L/v) \left(1 - \gamma^{-1} \right) = -\Delta t. \end{aligned}$$

- p.42, 2nd line above Eq.(3.2): Replace "2.4.2" by "2.3.2" so the sentence reads as "We have already shown in Section 2.3.2 that ..."

- p.46, Sidenote 11: Insert an equation reference "(12.17)" in the 3rd sentence so the Sidenote will read

  [11] A more formal...... Such a transformation, cf.(12.17), must satisfy the "generalized orthogonality condition",....

- p.49, above Eq.(3.39): Move Sidenote 15 to a new location, just above the displayed (3.39): ".... the relativistic energy as[15]", and insert a new sentence at the end of that sidenote: "... called the rest mass. In terms of the 4-velocity components, we have $p^i = mdr^i/d\tau$ and $E = mc^2 dt/d\tau$.

- p.52, the caption of Fig 3.3: Replace the last word "suppressed" by "displayed" so the sentence reads as "Invariant regions in the spacetime diagram, with two of the spatial coordinates displayed."

- p.53, the last line of the top paragraph: Replace the 1st word "large" by "larger".

- p.84, above Eq.(5.10): Insert a new Sidenote 3a at the end of the text prior to the displayed (5.10):

  [3a] Recall that a matrix multiplication involves the summation of a pair of row and column indices and that the indices a and b on g_{ab} are respectively the row and column indices.

- p.88, the line below Eq.(5.29): Insert a new Sidenote $6a$ so as "where[6a] we have used ..." with the new sidenote being

  [6a] We display the steps in the differentiation of a tensor in the first equation: Since we will differentiate with respect to $\dot{x}^a$ having index a, we should change the dummy indices in the sum $L(x, \dot{x}) = g_{ab}\dot{x}^{\,a}\dot{x}^b = g_{cb}\dot{x}^c\dot{x}^b$, leading to

$$\begin{aligned}\frac{\partial L}{\partial \dot{x}^{\,a}} &= \frac{\partial}{\partial \dot{x}^{\,a}}\left[g_{cb}\dot{x}^c\dot{x}^b\right] \\ &= g_{cb}\frac{\partial \dot{x}^{\,c}}{\partial \dot{x}^{\,a}}\dot{x}^b + g_{cb}\dot{x}^c\frac{\partial \dot{x}^b}{\partial \dot{x}^{\,a}} \\ &= g_{cb}\delta^c_a\dot{x}^b + g_{cb}\dot{x}^c\delta^b_a = 2g_{ab}\dot{x}^b.\end{aligned}$$

- p.109, 1^{st} line below Eq.(6.24): Insert two words "locally measured" before "frequency" so it will read as "On the other hand, the locally measured frequency being inversely proportional . . . "

- p.121, the 2^{nd} line below Eq.(7.16): Insert a new Sidenote $2a$ at the end of the sentence" "... in the time direction[2a]."

  [2a] In the Schwarzschild metric (7.16) to be discussed below, we note that for $r < r^*$ the elements change signs, $g_{rr} < 0$ and $g_{00} > 0$, so the space/time roles of the r and t coordinates switch. We will explore this more when we discuss black holes in Chapter 8."

- p.131, the line above Eq.(7.47): Insert a new Sidenote $14a$, "... which is particle energy[14a] E/mc, and, ..."

  [14a] In the SR limit of $r^* = 0$, it reduces to the energy $E = mc^2 dt/d\tau$ as shown in Sidenote 15 on p.49.

- p.149, the 4th line from the bottom: Insert a new Sidenote $5a$ "...towards the $r = 0$ line[5a] so that..."

  [5a] In the $r < r^*$ region the lightlike geodesics of (8.14) $cd\bar{t} = -dr$ and (8.22) $cd\bar{t} = dr\,(r + r^*)\,/\,(r - r^*)$ are both incoming as r decreases with an increasing $\bar{t}$.

- p.159, the 1st line of the second paragraph: Insert a new Sidenote $12a$ at the end of the line as in "... Oppenheimer and Snyder performed a GR study[12a] and made most...."

  [12a] Their research on gravitational collapse showed analytically that a cold Fermi gas quickly collapses from a smooth initial distribution to form a black hole with the properties discussed above.

- p.174, the 3rd line below Eq.(8.80): Insert a new Sidenote 25 at the end of the sentence "...area is ever-increasing[25]."

  [25] With BH area being proportional to squared mass, the initial area of a spherical mass dM falling into a black hole $(A^* + dA^*) \propto \left(M^2 + dM^2\right)$ is clearly less than the final state BH area $\propto (M + dM)^2$.

- p.197, Eq.(9.37): Insert a set of missing brackets [...]. It should appear as "$dl^2 = R_0^2 a^2(t)\left[d\chi^2 + k^{-1}\left(\sin^2\sqrt{k}\chi\right)d\Omega^2\right]$".

- p.204, Problem 9.11(b): Change in the text "Problem 9.10" to "Problem 9.9".

- p.216, Eq.(10.35): Replace h by $\hbar$ ("h bar") so as to have

$$n_b = \frac{4}{3}n_f = 2.404\frac{g}{2\pi^2}\left(\frac{k_B T}{\hbar c}\right)^3$$

- p.217, the line above Eq.(10.43): Insert a new Sidenote $8a$ at the end of the phrase "leading to[8a]"

  [8a] The effective spin degrees of freedom of (10.38) has the value here: $g^* = 2\,(photon) + \frac{7}{8} \times 4(e^+, e^-) + \frac{7}{8} \times 6(\nu, \bar{\nu}) = 10\frac{3}{4}$.

- p.219, 5^{th} line from the bottom of the top paragraph: Change the value from "$n_\nu \simeq 150$ cm^{-3}" to "$n_{\nu,0} \simeq 335$ cm^{-3}".

- p.221, Eq.(10.53): Change the value from "0.7 MeV" to "0.1 MeV" so the displayed equation reads as $k_B T_{\rm bbn} \simeq 0.1$ MeV,"

- p.226, Eq.(10.65): Replace h by $\hbar$ so that the equation reads as

$$n_{\gamma,0} = \frac{2.4}{\pi^2}\left(\frac{k_B T_{\gamma,0}}{\hbar c}\right)^3 \simeq 410/\text{cm}^3.$$

- p.226 - 227, the paragraph below Eq.(10.66): Modify the entire paragraph except the first sentence. Namely replace the sentences

  "This also explains why ... ... a new equilibrium phase."

  by the sentences

  "Considering that the average photon energy at decoupling is $2.7 \times 0.26 \simeq 0.70$ eV, why was the ionization not shut off until the thermal energy fell so much below the hydrogen ionization energy of 13.6 eV? This just reflects the fact that there are so many photons for every baryon that even though the average photon energy was only 0.70 eV, there remained up to that time a sufficient number of high energy photons at the tail end of the distribution to hold off the transition to a new equilibrium phase.".

- p.227, Eq.(10.67): Replace "$k_B T_{\gamma,0}$" in the denominator of the 2^{nd} line by "0.70 $eV/(1100)$", and replace the right hand side "8.8" by "3.3" so that the displayed equation reads as

$$\begin{aligned} \frac{\Omega_{M,0}}{\Omega_{R,0}} &= \frac{\Omega_{M,0}}{\Omega_{B,0}} \frac{\Omega_{B,0}}{1.68 \times \Omega_{\gamma,0}} \\ &\simeq \frac{0.25}{0.04} \frac{n_B}{n_\gamma} \frac{m_N c^2 \times 1100}{1.68 \times 0.70\ eV} \simeq 3.3 \times 10^3, \end{aligned} \qquad (10.67)$$

- p.227, 3^{rd} line below Eq.(10.67): Replace "$k_B T_{\gamma,0}$" by "0.70 $eV/(1100)$ as the average value was 0.70 eV at decoupling ($z = 1100$)" so that the sentences reads

  " .... and photon energy by 0.70 $eV/(1100)$ as the average value was 0.70 eV at decoupling ($z = 1,100$). The baryon photon ratio ...."

- p.227-228, Eq.(10.68) *and* the paragraph above and below Eq.(10.69): Replace in Eq.(10.68) the factor "1.1×10^4" by "3300", replace on the 1^{st} line below Eq.(10.68) the factor "8800" by "3300", on the 2^{nd} line replace the word "eight" by the word "three", on the 3^{rd} line replace the factor "8" by "3", and in Eq.(10.69) replace the numbers "8" by "3" and "2" by "1", respectively, as well as on 2^{nd} and 3^{rd} lines from the top of p.228, replace the number "16,000" by "70,000", and insert "t_{RM}" after "From that time" so that the new Eq.(10.68), Eq.(10.69) and the paragraphs that follow will read as "

$$1 = \frac{\rho_R}{\rho_M} = \frac{\rho_{R,0}}{\rho_{M,0}} [a(t_{RM})]^{-1} = \frac{\Omega_{R,0}}{\Omega_{M,0}} (1 + z_{RM}) \simeq \frac{1 + z_{RM}}{3300}. \qquad (10.68)$$

Hence the redshift for radiation–matter equality is $z_{RM} \simeq 3,300$, which is three times larger than the photon decoupling time with $z_\gamma \simeq 1,100$. This also yields scale factor and temperature ratios of $[a(t_\gamma)/a(t_{RM})] = [T_{RM}/T_\gamma] \simeq 3$, or a radiation thermal energy

$$k_B T_{RM} = 3\ k_B T_\gamma = O(1\ \text{eV}). \qquad (10.69)$$

Knowing this temperature ratio we can find the "radiation–matter equality time" $t_{RM} \simeq 70,000$ yrs (Problem 10.9) from the photon decoupling time $t_\gamma \simeq 360,000$ yrs. From that time t_{RM} on, gravity (less opposed by significant radiation pressure) began to grow, from the tiny lumpiness in matter distribution, the rich cosmic structures we see today."

- p.229, 3rd line below (10.75): Insert a new Sidenote 20a as in "... and neutrino number densities^{20a} as first stated in Section 10.3.2." The new sidenote reads as

  ^{20a} From the neutrinos' temperature, one can fix their number density via (10.35). Because neutrinos are fermions and photons are bosons, we have

  $$\frac{n_\nu}{n_\gamma} = \frac{3}{4}\left(\frac{T'_\nu}{T'_\gamma}\right)^3 \frac{g_\nu}{g_\gamma}$$

  As there are two photon states and six neutrino states (left-handed neutrinos and right-handed antineutrinos for each of the three lepton flavors). Plugging in these multiplicities and the temperature ratio from (10.75) yields $n_\nu/n_\gamma = 9/11$ or $n_{\nu,0} \simeq 335\ \mathrm{cm}^{-3}$.

- p.236, Problem 10.6, the 1st line on top: Replace the phrase "the average photon energy was $\bar{u} \simeq 0.26\ eV$" by "and the average thermal energy was $k_{\mathrm{B}} T_\gamma \simeq 0.26\ eV$."

- p.236, Problem 10.9, the end of 4^{th} line on: Replace the number "8800" by "3300".

- p.236, Problem 10.12, the displayed equation: Insert vertical bars around the factor "$\Omega_0 - 1$" so that it will read as

  $$R_0 = \frac{c}{H_0\sqrt{|\Omega_0 - 1|}}.$$

- p.236, Replace the entire Problem 10.13 by the new wordings:

  10.13 Cosmological limit of neutrino mass Eqs.(9.27) and (9.28) inform us that the dark matter density parameter being $\Omega_{DM} = \Omega_M - \Omega_B \simeq 0.21$, if we assume that this dark matter is composed entirely of light neutrinos and antineutrinos, what limit can be obtained for the average mass of neutrinos (averaged over three flavors ν_e, ν_μ, and ν_τ)?"

- p.239, 1st line below the displayed equation on top: Insert a minus sign "$-$" in the 2nd term of the inline equation so it reads as

  "where $T^\Lambda_{\mu\nu} = -\kappa^{-1}\Lambda g_{\mu\nu} = (-c^4\Lambda/8\pi G_{\mathrm{N}})g_{\mu\nu}$ can be interpreted ..."

- p.263, Sidenote 28: Insert a pair of missing square brackets in the equation's 1^{st} line and replace the equation's 2^{nd} line:

  $$\begin{aligned} t(a) &= t_{\mathrm{H}} \int_0^a [\Omega_{\mathrm{M},0}/a' + \Omega_\Lambda a'^2]^{-1/2} da' \\ &= \frac{2t_{\mathrm{H}}}{3\sqrt{\Omega_\Lambda}} \sin^{-1}\left[\frac{a^{3/2}}{(\Omega_{\mathrm{M},0}/\Omega_\Lambda)^{1/2}}\right]. \end{aligned}$$

- p.263, 3^{rd} line below Eq.(11.48): Change the number 7 to 8.6 so that part of the sentence reads as "... of $t_{\rm tr} = t(a = 0.56) \simeq 8.6\,{\rm Gyr}$—in cosmic..."

- p.275, Problem 11.3, the displayed equation: Delete a factor of c and move the factor H_0 out of the integral sign, so that the equation reads as

$$d_{\rm L}(z) = \frac{c(1+z)}{H_0}\int_0^z \frac{dz'}{\left[\Omega_{\rm M,0}(1+z')^3 + \Omega_\Lambda\right]^{1/2}}.$$

- p.275, Problem 11.5: Replace the problem by the new wordings:

  11.5 Estimate of matter and dark energy equality time Closely related to the deceleration/acceleration ("inflection") transition time $t_{\rm tr}$ is the epoch $t_{\rm M\Lambda}$ when the matter and dark energy components are equal $\Omega_{\rm M}(t_{\rm M\Lambda}) = \Omega_\Lambda(t_{\rm M\Lambda})$. Show that the estimated matter and dark energy equality time $t_{\rm M\Lambda}$ is comparable to, and as expected somewhat greater than, $t_{\rm tr} \simeq$ 8.6 Gyr of (11.48)."

- p.281, the paragraph below (12.6): After the 1st sentence revise the paragraph including the interchange of the words "contravariant" and "covariant" so that the paragraph reads as

  "Repeated indices are summed in the expansions of the vector **A**. However, as a shorthand, it is common practice to refer A_μ or A^ν as the vector **A** itself, even though A_μ and A^ν are technically the covariant and contravariant components, respectively, of the vector **A**. In this convention, A_μ is called a covariant vector and A^ν a contravariant vector."

- p.283, Sidenote 7: Add a sentence at the end of the sidenote, "Cf. also Sidenote 9a on p.287."

- p.287, above Eq.(12.40): Insert a new Sidenote $9a$ at the end of the text prior to the displayed (12.40):

  [9a] Recall comments made in Sidenote $3a$ on p.84.

- p.288, at the end of Box 12.1: Reverse the order of superscript indices ($\mu \leftrightarrow \nu$) on $F^{\mu\nu}$ in both Eqs.(12.41) and (12.42) so they read as

$$F^{\nu\mu}F_{\mu\nu} = 2\left(\vec{E}^2 - \vec{B}^2\right) \qquad (12.41)$$

  and

$$F^{\nu\mu}\tilde{F}_{\mu\nu} = 4\left(\vec{E}\cdot\vec{B}\right). \qquad (12.42)$$

- p.303, Eqs (13.26) and (13.27): Change the order of subscript indices $(\nu\lambda)$ to $(\lambda\nu)$ in three places so as to have

$$\partial_\nu \mathbf{e}^\mu = -\Gamma^\mu_{\lambda\nu}\mathbf{e}^\lambda \qquad \text{or} \qquad \mathbf{A}\cdot(\partial_\nu \mathbf{e}^\mu) = -\Gamma^\mu_{\lambda\nu}A^\lambda, \tag{13.26}$$

and

$$D_\nu A^\mu = \partial_\nu A^\mu + \Gamma^\mu_{\lambda\nu}A^\lambda. \tag{13.27}$$

- p.311, Eq.(13.57): Insert a minus sign on the right hand side

$$dA^\mu = -R^\mu{}_{\nu\lambda\rho}A^\nu a^\lambda b^\rho. \tag{13.57}$$

and add a new Sidenote *8a* at the end of the text above the displayed equation (13.57):

[8a] The minus sign is required so as to be compatible with the curvature definition given in (13.58), if the direction of the parallel transport loop of Box 13.2 is in accord with the area direction (13.56), *i.e.*, given by the right-hand rule around $\boldsymbol{\sigma}$ in the 2D case (counterclockwise in Fig. 13.5).

- p.312, Eq.(13.65): Interchange the order of subscript indices $(\alpha \leftrightarrow \beta)$ in the 2^{nd} line on the right hand side of the displayed equation so it will read as

$$\begin{aligned} dA^\mu_{PQ'P'} &= -\Gamma^\mu_{\nu\alpha}A^\nu b^\alpha - \Gamma^\mu_{\nu\beta}A^\nu a^\beta + A^\nu\Gamma^\mu_{\nu\beta}\Gamma^\beta_{\rho\sigma}a^\rho b^\sigma \\ &\quad -\partial_\beta\Gamma^\mu_{\lambda\alpha}A^\lambda a^\alpha b^\beta + \Gamma^\mu_{\nu\alpha}\Gamma^\nu_{\lambda\beta}A^\lambda a^\alpha b^\beta. \end{aligned} \tag{13.65}$$

- p.312, Eq.(13.66): Interchange the order of two terms on the 1st line and insert a minus sign on the 2nd line so that the equation will read as

$$\begin{aligned} dA^\mu &= dA^\mu_{PQP'} - dA^\mu_{PQ'P'} \\ &= -\left[\partial_\alpha\Gamma^\mu_{\lambda\beta} - \partial_\beta\Gamma^\mu_{\lambda\alpha} + \Gamma^\mu_{\nu\alpha}\Gamma^\nu_{\lambda\beta} - \Gamma^\mu_{\nu\beta}\Gamma^\nu_{\lambda\alpha}\right]A^\lambda a^\alpha b^\beta \end{aligned} \tag{13.66}$$

- p.313, the line above (13.67): Insert a new Sidenote 9*a* at the end of the text prior to the displayed (13.67):

[9a] The substitution rule (14.1) $d_x \to D_x$ will help us to see how the expression of curvature by the commutator of covariant derivatives is related to that by transport a vector around an infinitesimal closed path performed in Box 13.2. The displacement of a vector A by dx in a curved space may be written schematically as $A(x+dx) = A + dxD_xA = (1+dxD_x)A$ so that the transport of this vector around a closed parallelogram spanned by dx and dy can then be expressed as $(1+dxD_x)(1+dyDy)A - (1+dyD_y)(1+dxD_x)A = [D_x, D_y]Adxdy$.

- p.317, Problem 13.11: Remove 2 minus signs so that Part (a) will have $K = R_{1212}/g$ and Part (b) have $R = 2K$.

- p.325, Eq.(14.32): Remove the minus sign from the first displayed equation so it reads as
$$\Gamma^0_{00} = \frac{\dot{\nu}}{2}$$

- p.326, the 3rd displayed equation from the bottom of the page: Change the minus to a plus sign for the middle term so it reads as
$$-e^{\nu}\left[\frac{d^2x^0}{d\sigma^2} + \nu'\frac{dr}{d\sigma}\frac{dx^0}{d\sigma} + \frac{\dot{\nu}}{2}\left(\frac{dx^0}{d\sigma}\right)^2 + \frac{\dot{\rho}}{2}e^{\rho-\nu}\left(\frac{dr}{d\sigma}\right)^2\right] = 0$$

  Drop the minus sign on the RHS of the middle of Eq.(14.37) so it reads as
$$\Gamma^0_{00} = \frac{\dot{\nu}}{2},$$

- p.335, Problem 14.4, Eq.(14.75): Replace the letter "D" in the left hand side denominator so that the eqn reads $\frac{D^2 s^\mu}{d\tau^2} = -R^\mu{}_{\alpha\nu\beta} s^\nu \frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau}$

- p.335, Problem 14.6, Eq.(14.76): Insert the power 2 on the LHS equation so that it reads
$$\Omega^2 = \frac{G_\mathrm{N} M}{R^3} = \frac{r^*}{2R^3}$$

- p.337, 2nd to the last line of the first paragraph (below the bullet box): Insert Sidenote $1a$ at "...in a background of flat spacetime[1a]. It is a transverse..." with the new sidenote being

  [1a] For an insightful discussion of describing gravitational phenomena using the curved spacetime versus the flat spacetime paradigms, see p.403 of (Thorne 1994).

- p.340-341, Eq.(15.20) and Table 15.1: Insert a minus sign so as to have $\Box\bar{h}_{\mu\nu} = -\frac{16\pi G_\mathrm{N}}{c^4}T^{(0)}_{\mu\nu}$ in (15.20) and in the last row of Table 15.1. Also insert a minus sign and remove 4π so as to have $\Box A_\mu = -\frac{1}{c}j_\mu$ in the middle of that row.

- p.343, Eq.(15.30): Change the + sign into a subscript of the letter h so that it reads as "$= \left[1 + \frac{1}{2}h_+ \sin\omega\,(t - z/c)\right]\xi$."

- p.344, 1st sentence below Eq.(15.31): Change the sentence from "Thus, the separation ... ... compressed." to

  "Thus, the separation between two test masses is determined to be an elongation in x direction and a compression in the y direction."

- p.344, 3rd line in the 2nd paragraph "The effect of a wave ...":

  Change the $\times$ sign into a subscript of the letter h so that it reads as "... $ds = [1 \pm \frac{1}{2} h_\times \sin \omega(t - z/c)]\xi$."

- p.349, the line above (15.44) as well as Eqs.(15.44) and (15.45) themselves: Insert the clause ", after dropping higher order terms in $\tilde{h}_+$," just ahead of the last word "are" on the line above the displayed equation (15.44), and drop the 2nd terms in the parenthesis of (15.44) and (15.45) so that these two equations with the revised line above them will read as

  "... elements, after dropping higher order terms in $\tilde{h}_+$, are

  $$\Gamma^1_{10} = \Gamma^1_{01} = \Gamma^0_{11} = \frac{1}{2}\partial_0 \tilde{h}_+ \tag{15.44}$$

  and, similarly,

  $$\Gamma^1_{13} = \Gamma^1_{31} = -\Gamma^3_{11} = -\frac{1}{2}\partial_0 \tilde{h}_+ \tag{15.45}$$

- p.382, Problem 6.2: In 2nd to the last line insert a "slash" in front of g_{00} in the inline equation so it reads as "$\gamma_{ij} = g_{ij} - (g_{0i} g_{0j}/g_{00})$".

- p.385, Problem 7.3: the 3rd displayed equation

  $$(u_0'' + u_0) + \epsilon \left(u_1'' + u_1 - u_0^2\right) + ... = 0$$

  - the line above this equation replace the factor $O(r^*)$ by $3r^*/2$ so the last inline equation reads as "with $\epsilon = 3r^*/2$,"
  - 2nd line below this displayed equation, change $\sin\phi$ to $cos\phi$ so the inline equation reads as $u_0 = r_{\min}^{-1} \cos\phi$
  - Change the sign in front of $\cos 2\phi$ in the following displayed equation so that

    $$\frac{d^2 u_1}{d\phi^2} + u_1 = \frac{1 + \cos 2\phi}{2r_{\min}^2}$$

  - the line below this equation insert a minus sign in the expression $\beta = -\left(6r_{\min}^2\right)^{-1}$ and in the following displayed equation
  - so it becomes, after change $\sin\phi$ to $cos\phi$ on the 1st term on the RHS,

    $$\frac{1}{r} = \frac{\cos\phi}{r_{\min}} + \frac{3 - \cos 2\phi}{4} \frac{r^*}{r_{\min}^2}.$$

– Replace the paragraph and the displayed equation below by "In the absence of gravity ($r^* = 0$), the asymptotes ($r = \infty$) corresponds to initial and final angles of $\phi_i = \pi/2$ and $\phi_f = -\pi/2$, and the trajectory is straight line (no deflection). When gravity is turned on, there is an angular deflection $\phi_i = \pi/2 + \delta\phi/2$ and $\phi_f = -\pi/2 - \delta\phi/2$, and the trajectory equations yields (for either asymptote):

$$0 = -\sin\frac{\delta\phi}{2} + \frac{3 - \cos\pi}{4}\frac{r^*}{r_{\min}}$$

For small deflection angle $\delta\phi$,

$$0 = -\frac{\delta\phi}{2} + \frac{r^*}{r_{\min}};$$

we obtain the result of $\delta\phi_{\mathrm{GR}} = 2r^*/r_{\min}$." (This should be the end of Prob 7.3.)

- p.395, Problem 10.7, 5th line from the top: Delete the word "negative" and the parentheses so that it reads as "... parameter is $w = -1, ...$".

- p.395, Problem 10.9: in the displayed equation, move the factors of "t_γ" ahead of the brackets, and replace the number "$8^{3/2}$" by "$3^{3/2}$", and "16 000" by "70 000" so the displayed equation becomes

$$t_{\mathrm{RM}} = t_\gamma\left[\frac{a(t_{\mathrm{RM}})}{a(t_\gamma)}\right]^{3/2} = t_\gamma\left[\frac{1 + z_{\mathrm{RM}}}{1 + z_\gamma}\right]^{-3/2} \simeq \frac{t_\gamma}{3^{3/2}} \simeq 70,000 \text{ years.}$$

- p.395, Problem 10.13, Replace the entire solution by

10.13 Cosmological limit of neutrino mass If we assume that dark matter is made up of neutrinos $\rho_{\mathrm{DM}} = n_\nu\bar{m}$, where $\bar{m}$ is the average neutrino mass. The neutrino number density n_ν includes the three flavors ν_e, ν_μ, ν_τ,and their antiparticles. We now relate n_ν to photon densities $n_\gamma$by way of (10.35). First we note that the polarization degrees of a photon $g_\gamma = 2$ and that each light neutrino has only one helicity state, for three flavors of neutrinos and antineutrinos, a total light neutrino degrees of freedom are $g_\nu = 6$ Furthermore neutrino being a fermion and photon a boson, there is a factor of 4/3 difference in their density ratio as shown in (10.35): $n_\nu = \frac{6}{2} \times \frac{3}{4}\left(\frac{T_\nu}{T_\gamma}\right)^3 n_\gamma$. From the neutrino and photon temperature ratio of $T_\gamma = (11/4)^{1/3}\,T_\nu$ obtained in (10.75) and photon number density of $n_\gamma \simeq 410$ cm^{-3} as derived in (10.65), we then have $n_\nu = \frac{9}{4}\left(\frac{4}{11}\right) \times 410 \simeq$ 335 cm^{-3}. If we identify the neutrino density parameter to the dark matter density $n_\nu\bar{m}c^2/\left(\rho_{\mathrm{c}}c^2\right) = \Omega_{\mathrm{DM}} \simeq 0.21$, we then obtain the constraint on the average neutrino mass $\bar{m}c^2 \simeq 0.21\times 5,500/340 \simeq 3.4$ eV,where we have used the critical energy density $\rho_c c^2$ value as given in (9.17). This $\bar{m}c^2$ value is for the total neutrino and antineutrino masses for all flavors; per neutrino of a given flavor, we have the average value of $\left(\bar{m}_\nu c^2\right)_1 \simeq 0.6$ eV

- p.396, Problem 11.4: Insert the three missing square brackets in all three lines of the displayed equations so that the entire revised solution 11.4 reads as

11.4 Negative Λ and the "big crunch" For the $\Omega_0 = 1$ flat universe with matter and dark energy, we have the Friedmann (11.38)

$$H(a) = H_0[\Omega_{\mathrm{M},0}a^{-3} + \Omega_\Lambda]^{1/2}.$$

At $a = a_{\max}$ the universe stops expanding and $H(a_{\max}) = 0$, thus $a_{\max} = (-\Omega_{\mathrm{M},0}/\Omega_\Lambda)^{1/3}$. The cosmic time for the big crunch being twice the time for the universe to go from $a_{\max}$ to $a = 0$, we calculate in a way similar to that shown in sidenote 28 on p,263

$$2t_{\mathrm{H}} \int_0^{a_{\max}} \frac{da}{[\Omega_{\mathrm{M},0}a^{-1} + \Omega_\Lambda a^2]^{1/2}} = \frac{4t_{\mathrm{H}}}{3\sqrt{-\Omega_\Lambda}} \int_0^{a_{\max}^{3/2}} \frac{dx}{[a_{\max}^3 - x^2]^{1/2}}$$
$$= \frac{4t_{\mathrm{H}}}{3\sqrt{-\Omega_\Lambda}} \left[\sin^{-1}\left(\frac{x}{a_{\max}^{3/2}}\right)\right]_0^{a_{\max}^{3/2}} = \frac{2\pi}{3\sqrt{-\Omega_\Lambda}} t_{\mathrm{H}} = t_*.$$

- p.396, Problem 11.5: Replace the solution by

11.5 Estimate of matter and dark energy equality time We define the matter and dark energy equality time $t_{\mathrm{M}\Lambda}$ as $\rho_{\mathrm{M}}(t_{\mathrm{M}\Lambda}) = \rho_\Lambda(t_{\mathrm{M}\Lambda})$.Using the scaling properties of these densities we have $\rho_{\mathrm{M},0}/a_{\mathrm{M}\Lambda}^3 = \rho_{\Lambda,0}$ or $a_{\mathrm{M}\Lambda}^3 = \Omega_{\mathrm{M},0}/\Omega_\Lambda$, which differs from the deceleration-acceleration transition scale factor result [cf. (11.47) $a_{\mathrm{tr}}^3 = \Omega_{\mathrm{M},0}/2\Omega_\Lambda$ by a factor of $2^{1/3} \approx 1.25$ to yield $a_{\mathrm{M}\Lambda} = 0.7$ (and a redshift of $z_{\mathrm{M}\Lambda} = 0.42$)] . Plugging in $a = a_{\mathrm{M}\Lambda} = (\Omega_{\mathrm{M},0}/\Omega_\Lambda)^{1/3}$ into the formula given in Sidenote 28, we obtain the corresponding cosmic age $t_{\mathrm{M}\Lambda} = t\,(a_{\mathrm{M}\Lambda}) = 9.5$ Gyr."

- p.405, Problem 13.11(a): Remove an extra factor of 1 in the subscript of the denominator factor g_{221} so that them 4th displayed equation line reads as

$$\Gamma^2_{22} = \frac{1}{2g_{22}}\partial_2 g_{22}, \qquad \Gamma^2_{12} = \Gamma^2_{21} = \frac{1}{2g_{22}}\partial_1 g_{22}$$

- p.405, Problem 13.11(a): Insert 3 minus signs in front of the last three right hand sides and remove an extra factor of ν in the subscript of $\Gamma^1_{1\nu 1}$

on the 3rd line so that the last displayed equation on p.405 reads as

$$\begin{aligned}
& R_{1212} = g_{1\mu} R^{\mu}_{212} \\
= & -g_{11}\left(\partial_2 \Gamma^1_{21} - \partial_1 \Gamma^1_{22} + \Gamma^{\nu}_{21}\Gamma^1_{\nu 2} - \Gamma^{\nu}_{22}\Gamma^1_{\nu 1}\right) \\
= & -g_{11}(\partial_2 \Gamma^1_{21} - \partial_1 \Gamma^1_{22} + \Gamma^1_{21}\Gamma^1_{12} + \Gamma^2_{21}\Gamma^1_{22} - \Gamma^1_{22}\Gamma^1_{11} - \Gamma^2_{22}\Gamma^1_{21}) \\
= & -\frac{1}{2}\{\partial_2^2 g_{11} + \partial_1^2 g_{22} - \frac{1}{2g_{11}}\left[\left(\partial_1 g_{11}\right)\left(\partial_1 g_{22}\right) + \left(\partial_2 g_{11}\right)^2\right] \\
& -\frac{1}{2g_{22}}\left[\left(\partial_2 g_{11}\right)\left(\partial_2 g_{22}\right) + \left(\partial_1 g_{22}\right)^2\right]\}
\end{aligned}$$

- p.405, Problem 13.11(a) and (b) the last five lines on p.405: Remove two minus signs: one in front of R_{1212}/g on the last 4th line and the other in front of $2K$ on the last line so that these five lines now read

  "which, when divided by the metric determinant $g = g_{11}g_{22}$, the ratio R_{1212}/g is recognized as the Gaussian curvature of (5.35).

  (b) The Ricci scalar is simply the twice contracted Riemann tensor $R = g^{\alpha\beta}g^{\mu\nu}R_{\alpha\mu\beta\nu} = 2g^{11}g^{22}R_{1212}$ because $R_{1212} = R_{2121}$. Since $g^{11} = 1/g_{11}$ and $g^{22} = 1/g_{22}$, $g^{11}g^{22} = 1/g$ for a diagonalized metric tensor. In this way the result (a) leads to $R = 2K$."

- p.406, Problem 13.11(c) the top line and the top displayed equation: Insert a minus sign in the inline equation $dA^2 = R^2{}_{112}A^1\sigma$, a minus sign after the 1st equal sign and remove a minus sign after the 2nd equal sign of the displayed equation so that the text and the displayed equation now read

  (c) Eq (13.57) may be written as $dA^2 = -R^2{}_{112}A^1\sigma$. Since the angular excess is related to the vector component change as $\epsilon = dA^2/A^1$, we have

$$\begin{aligned}
\epsilon &= -R^2{}_{112}\sigma = g^{22}R_{1212}\sigma = g^{22}gK\sigma \\
&= g^{22}\left(g_{11}g_{22}\right)K\sigma = K\sigma,
\end{aligned}$$

- p.406, Problem 13.12(2) the right hand side of the 2nd to the last displayed equation: insert two sets of parentheses on the 3^{rd} and 4^{th} line so the displayed equation reads as

$$\begin{aligned}
\left[D_\lambda, \left[D_\mu, D_\nu\right]\right] A_\alpha &= D_\lambda \left[D_\mu, D_\nu\right] A_\alpha - \left[D_\mu, D_\nu\right] D_\lambda A_\alpha \\
&= -D_\lambda\left(R^{\gamma}{}_{\alpha\mu\nu}A_\gamma\right) + R^{\gamma}{}_{\alpha\mu\nu}D_\lambda A_\gamma + R^{\gamma}{}_{\lambda\mu\nu}D_\gamma A_\alpha \\
&= -\left(D_\lambda R^{\gamma}{}_{\alpha\mu\nu}\right)A_\gamma - R^{\gamma}{}_{\alpha\mu\nu}D_\lambda A_\gamma + R^{\gamma}{}_{\alpha\mu\nu}D_\lambda A_\gamma + R^{\gamma}{}_{\lambda\mu\nu}D_\gamma A_\alpha \\
&= -\left(D_\lambda R^{\gamma}{}_{\alpha\mu\nu}\right)A_\gamma + R^{\gamma}{}_{\lambda\mu\nu}D_\gamma A_\alpha,
\end{aligned}$$

- p.406, Problem 13.12(2) the right hand side of the last displayed equation: (i) insert three sets of parentheses on the 2^{nd} line, (ii) change two minus signs in the 4^{th} line so the displayed equation reads as

$$\begin{aligned} 0 &= [D_\lambda,[D_\mu,D_\nu]] + [D_\nu,[D_\lambda,D_\mu]] + [D_\mu,[D_\nu,D_\lambda]] \\ &= -\left(D_\lambda R^\gamma{}_{\alpha\mu\nu}\right) A_\gamma - \left(D_\nu R^\gamma{}_{\alpha\lambda\mu}\right) A_\gamma - \left(D_\mu R^\gamma{}_{\alpha\nu\lambda}\right) A_\gamma \\ &\quad + R^\gamma{}_{\lambda\mu\nu} D_\gamma A_\alpha + R^\gamma{}_{\nu\lambda\mu} D_\gamma A_\alpha + R^\gamma{}_{\mu\nu\lambda} D_\gamma A_\alpha \\ &= -\left(D_\lambda R^\gamma{}_{\alpha\mu\nu} + D_\nu R^\gamma{}_{\alpha\lambda\mu} + D_\mu R^\gamma{}_{\alpha\nu\lambda}\right) A_\gamma \\ &\quad + \left(R^\gamma{}_{\lambda\mu\nu} + R^\gamma{}_{\nu\lambda\mu} + R^\gamma{}_{\mu\nu\lambda}\right) D_\gamma A_\alpha. \end{aligned}$$

- p.411, solution to Problem 15.2, the equation at the bottom of the page: Insert a missing power of 3 over r in the very last factor so that the factor changes from "$\partial^\mu \bar{h}_{\mu\nu} = ... \quad = -\frac{x^i C_{i\nu}}{r} = 0$." to

$$\partial^\mu \bar{h}_{\mu\nu} = C_{\mu\nu}\partial^\mu \left(\frac{1}{r}\right) = C_{i\nu}\partial^i \left(\frac{1}{r}\right) = -\frac{x^i C_{i\nu}}{r^3} = 0.$$

- p.413, Problem 15.4(b): Insert the parenthetical remark so the first two lines of text just above the displayed equations will read as

  (b) Ricci tensor: from what we know of Christoffel symbols having the nonvanishing elements of (after dropping higher order terms in $\tilde{h}_+$)