HW #12

3. Show that the line element $ds^2 = y^2dx^2 + x^2dy^2$ represents the Euclidean plane, but the line element $ds^2 = ydx^2 + xdy^2$ represents a curved two-dimensional space. (Hint: Calculate the elements of the metric tensor, the Christoffel symbols, and then the Riemann curvature tensor. From the latter, calculate the Gaussian curvature K as noted at the top of page 314. (Note: There are two typographical erros at the top of page 314. The correct relationships are $K = \frac{1}{g}R_{1212}$ and $R = 2K$.)

(4.) A spacetime has the metric

$$ds^2 = -c^2dt^2 + a^2(t)(dx^2 + dy^2 + dz^2).$$

Show that the only non-zero connection coefficients are

$$\Gamma^0_{11} = \Gamma^0_{22} = \Gamma^0_{33} = a\dot{a} \quad \text{and} \quad \Gamma^1_{10} = \Gamma^2_{20} = \Gamma^3_{30} = \dot{a}/a.$$

Show that particles may be at rest in such a spacetime and that for such particles the coordinate t is their proper time. What is the time dependence of the the separation ds^2 for two particles initially separated by some distance in space?

Show that the non-zero components of the Ricci tensor are

$$R_{00} = -3\ddot{a}/a \quad \text{and} \quad R_{11} = R_{22} = R_{33} = a\ddot{a} + 2\dot{a}^2.$$

Hence show that the 00-component of the Einstein tensor is $G_{00} = 3\dot{a}^2/a^2$.
(Note also that the dot notation used here means $\dot{a} = \frac{\partial a}{\partial x^0}$.)