Homework #4

1. Photons of frequency ν_E are emitted from the surface of the Sun and observed by an astronaut with fixed spatial coordinates at a large distance away. Obtain an expression for the frequency ν_O of the photons as measured by the astronaut. Hence, estimate the observed redshift of the photon.

2. An experimenter A drops a pebble of rest mass m in a uniform gravitational field g. At a distance h below A, experimenter B converts the pebble (with no energy loss) into a photon of frequency ν_B. The photon passes A, who observes it to have frequency ν_A. Use simple physical arguments to show that to a first approximation

$$\frac{\nu_B}{\nu_A} = 1 + \frac{gh}{c^2}.$$

Use this result to argue that for two stationary observers A and B in a weak gravitational field with potential Φ, the ratio of the rates at which their laboratory clocks run is $1 + \frac{\Delta\Phi}{c^2}$, where $\Delta\Phi$ is the potential difference between A and B.

3. A satellite in a circular polar orbit of radius r around the Earth (radius R, mass M). A standard clock C on the satellite is compared with an identical clock C_0 at the south pole on Earth. Show that the ratio of the rate of the orbiting clock to that of the clock on Earth is approximately

$$1 + \frac{GM}{Rc^2} - \frac{3GM}{2rc^2}.$$

Note that the orbiting clock is faster only if $r > \frac{3}{2}R$, i.e., if $r - R > 3184$ km. This is a simple model for the corrections needed in the GPS system.

4. A sodium lamp emits light in its rest frame with a wavelength of 5890 Å. If the lamp is placed on a turntable and is rotating with a speed of 0.2 c, what wavelength would an observer fixed at the center of the turntable measure?

(5). Using the Principle of Equivalence we derived a relationship between the frequency of a photon and the value of the gravitational potential it experiences:

$$\frac{\omega_1 - \omega_2}{\omega_2} = -\frac{\Phi_1 - \Phi_2}{c^2.}$$

This is valid if the change in Φ is small. However, if the change is large, then we should rewrite our result as

$$\frac{d\omega}{\omega} = -\frac{d\Phi}{c^2.}$$

a)Integrate this equation to find the relationship between the frequencies at two different locations in the gravitational potential.

b) Suppose that the Earth's density were to increase by a factor of 10^9. Show that a clock at the surface of the Earth would run at one-half the rate of one at infinity.