1. Show that $\left(\frac{1+z^2}{1-z}\right)_+ = \frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(1-z)$.

2. a) Using the DGLAP equation for the quark PDF, derive a lowest order expression for $\frac{\partial F_2(x,Q^2)}{\partial \ln Q^2}$.

b) At small x show that this is dominated by the gluon PDF.

c) When fitting data show that α_s is anticorrelated with the gluon PDF, *i.e.*, larger glue gives a smaller α_s and vice versa.

d) $\frac{\partial F_2(x,Q^2)}{\partial \ln Q^2}$ changes sign around $x \approx 0.2$. Show the origin of this sign change. Physically, why does this occur?

3 a) Write the DGLAP equation for $q_v(x,Q^2) = q(x,Q^2) - \bar{q}(x,Q^2)$ and show that the gluon doesn't contribute.

b) The moments of $A(x)$ are defined as $A^n = \int_0^1 dx x^{n-1} A(x)$. Take moments of both sides of the equation for $q_v(x,Q^2)$ and show that the convolution is factorized into a simple product of moments.

c) Solve the resulting equation in moments space using as boundary conditions the moments of q_v at $Q^2 = Q_0^2$. Note that if you were to do an inverse Mellin transform on the results you would have $q_v(x,Q^2)$ for any desired values of x and Q^2.

4. Consider the quark-quark contribution to high-p_T jet production denoted by σ. Show that by including the $\mathcal{O}(\alpha_s^3)$ NLO contributions one has $\mu^2 \frac{\partial \sigma}{\partial \mu^2} = 0 + \mathcal{O}(\alpha_s^4)$ and similarly for $M_f^2 \frac{\partial \sigma}{\partial M_f^2}$. Here μ is the renormalizations scale appearing in α_s while M_f is the factorization scale appearing inthe PDFs. See Appendix A of hep-ph/0303013 at arxiv.org but don't just copy the results - explain them!