PHY 5355: High-Energy Physics II, Fall 2015

Assignment # 2

due November 20, 2015

Introduction

The two problems presented in this Homework have been introduced in class and will serve to consolidate some aspects of Higgs-boson physics through calculations that will help you review all concepts discussed in class and apply them to new contexts.

While the quantum-field-theory principles on which Problem 1 and 2 rely (from gauge theories, to spontaneously broken gauge theories and their renormalization, to effective field theories) can be found in several textbooks, the applications discussed in Problem 1 and 2 are normally not textbook material. References to the literature will be given in each Problem. Part if not all the answers to the questions of Problem 1 and 2 are indeed retrievable in the literature if you study it carefully. I would like you to make good use of the results that you might be able to find: derive them, understand them, and show that you have mastered the content of each problem.

The deadline has been set in such a way that I can read your solutions and have the time to discuss them with you. If you think that it will be beneficial to have one or more discussion sessions before the due date, I will be glad to schedule them. However, since quite some care has been taken in explaining the problems, I expect you to come to the discussion session(s) having studied the material necessary to elaborate about each problem and having already made progress in each of them. You are also welcome to address your questions to me individually.

Problem 1

We have reviewed in class how the $SU(2)_L \times U(1)_Y$ gauge symmetry of the Standard Model (SM) is spontaneously broken to $U(1)_{\rm e.m.}$ via the Higgs mechanism. You can find all the core material in your QFT textbook(s) (for instance Peskin and Schroeder's or Srednicki's books), and a more expanded discussions as well as the material discussed in this problem in some TASI Lectures (see e.g. [1, 2] and references therein). Several extensions of the Standard Model include enlarged Higgs sectors, the minimal extension being the case of Two Higgs-Doublet Models (2HDM). The most general Higgs potential that can be constructed with two complex Higgs $SU(2)_L$ doublets of equal hypercharges ($Y = 1/2$), Φ_1 and Φ_2, subject to conditions of renormalizability and hermicity, is

$$\begin{aligned} V &= m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 + (m_{12}\Phi_1^\dagger\Phi_2 + \text{h.c.}) \\ &\quad +\lambda_1(\Phi_1^\dagger\Phi_1)^2 + \lambda_2(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) \\ &\quad +\left\{\frac{1}{2}\lambda_5(\Phi_1^\dagger\Phi^2)^2 + \left[\lambda_6(\Phi_1^\dagger\Phi_1) + \lambda_7(\Phi_2^\dagger\Phi_2)\right](\Phi_1^\dagger\Phi_2) + \text{h.c.}\right\}\ , \end{aligned} \tag{1}$$

where Φ_1 and Φ_2 can be represented in terms of their real components ϕ_i as,

$$\Phi_1 = \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix} \quad , \quad \Phi_2 = \begin{pmatrix} \phi_5 + i\phi_6 \\ \phi_7 + i\phi_8 \end{pmatrix}\ . \tag{2}$$

The potential parameters m_{11}^2, m_{22}^2, λ_1, λ_2, λ_3, and λ_4 are real, while m_{12}, λ_5, λ_6, and λ_7 are in general complex.

1.a) Explain why m_{11}^2, m_{22}^2, λ_1, λ_2, λ_3, and λ_4 have to be real and why, if the potential is CP invariant, the parameters m_{12}, λ_5, λ_6, and λ_7 have to real too. Justify why you would require the potential to be CP invariant.

1.b) Show that the conditions for the potential to be bounded from below are:

$$\lambda_1 \geq 0 \ , \quad \lambda_2 \geq 0 \ , \quad \lambda_3 + \lambda_4 + \lambda_5 \leq 2\sqrt{\lambda_2 \lambda_2} \ . \tag{3}$$

A possible way to tackle the problem is to examine the quartic terms of the quartic potential in terms of $a = \Phi_1^\dagger \Phi_1$, $b = \Phi_2^\dagger \Phi_2$, $c = \mathrm{Re}(\Phi_1^\dagger \Phi_2)$, and $d = \mathrm{Im}(\Phi_1^\dagger \Phi_2)$, and note that $ab \geq c^2 + d^2$. One then demands that no directions exist in field space in which $V \to -\infty$. This is clearly not the only way to proceed and you are free to come up with an alternative method.

1.c) Explain why a generic unconstrained 2HDM gives origin to flavor-changing neutral currents (FCNC) in the quark sector, and how this can be avoided by imposing a discrete Z_2 symmetry like,

$$\Phi_1 \to \Phi_1 \quad , \quad \Phi_2 \to -\Phi_2. \tag{4}$$

which also implies $m_{12} = \lambda_6 = \lambda_7 = 0$.

1.d) Upon spontaneous symmetry breaking (SSB), the SM with only one scalar doublet has the nice feature to generate the correct pattern of masses for the $W^\pm$ and the Z bosons, leaving the photon massless. In general, this is not the case anymore in extended models like the 2HDM: only particular choices of the parameters of the scalar potential keep the photon massless. As you will see, this forces an *alignement* of the vacuum expectation values (VEV).

The condition to have a massless photon is that the electric charge operator (Q) annihilates the VEV, namely $Q < \Phi_i >= 0$ ($i = 1, 2$). In the case of the SM with only one doublet we have seen that the most general VEV can be reduced, by performing an $SU(2)_L$ (gauge) rotation, to the form $< \Phi >^T = (0, v)$ (modulus a normalization factor $1/\sqrt{2}$). Analogously, in a 2HDM the most general VEV can be reduced, after $SU(2)_L \times U(1)_Y$ (gauge) rotation, to the form

$$< \Phi_1 >^T = (0, v_1) \quad , \quad < \Phi_2 >^T = (u_2, v_2 e^{i\xi}) \ , \tag{5}$$

where v_1, u_2, v_2, and ξ are real (with $v_1 > 0$). How can you further constrain the form of $< \Phi_1 >$ and $< \Phi_2 >$ to assure that the photon is massless?

1.e) Assuming that $m_{12} = \lambda_6 = \lambda_7 = 0$ (CP-conserving potential), derive the minimization conditions for the potential V (i.e. derive the eight conditions $\left.\frac{\partial V}{\partial \phi_i}\right|_{VEV} = 0$) and discuss how only the following alternatives are available:

$$\begin{array}{lll} (i) & < \Phi_1 >^T = (0, v_1) \ , & < \Phi_2 >^T = (0, v_2) \ , \\ (ii) & < \Phi_1 >^T = (0, v_1) \ , & < \Phi_2 >^T = (0, i v_2) \ , \\ (iii) & < \Phi_1 >^T = (0, v_1) \ , & < \Phi_2 >^T = (u_2, 0) \ . \end{array} \tag{6}$$

Explain why case (iii) is not allowed if you require a massless photon, and why (i) and (ii) are related.

1.f) Study the scalar mass spectrum of a 2HDM in case (i), (ii), and (iii) above. In order to do that you will have to expand the scalar potential about the minimum of the potential, i.e. in terms of fields $\Phi_i' = \Phi_i - <\Phi_i>$ ($i = 1, 2$), derive the mass matrix of the scalar fields, and diagonalize it by finding eigenvalues and eigenvectors. The eigenvectors represent the scalar fields that, upon SSB, have a definite mass term in the 2HDM Lagrangian (namely in the 2HDM scalar potential V). Notice that the (8×8) mass matrix of the 2HDM scalar sector has a very particular block-diagonal form that can greatly reduce the difficulty of your task. Given the relation between case (i) and (ii), only one of them needs to be studied. Case (iii) is on the other hand profoundly different and needs to be studied separately.

Discuss your result identifying the nature of each scalar mass eigenstate that you have found (scalar vs pseudoscalar, neutral vs charged, physical scalars vs Nambu-Goldstone bosons, etc.). In order to confirm and support your findings calculate the gauge-boson mass matrix and study the mass eigenstates. Your findings should be consistent with what you derived from the study of the scalar potential. Make sure to clearly explain the main differences between the scenario of case (i) and (ii) versus the scenario of case (iii).

From the form of the mass spectrum it should be clear that you can now draw a *phase diagram* of SSB that illustrate regions (i), (ii), and (iii) in the (λ_4, λ_5) parameter space (plane). How does it look like?

1.g) **Extra credit**. If $m_{12} \neq 0$ the original discrete Z_2 symmetry imposed to avoid FCNC is said to be *softly* broken (technically: broken by terms of dimension 2). Since FCNC are still absent at tree level, this could be a viable scenario. Repeat the exercise of points (**1.d**)-(**1.f**) for $m_{12} \neq 0$ and highlight the most important consequences.

Problem 2

We have discussed in class how the effects of new physics (NP) beyond the Standard Model (SM) on SM observables can be systematically studied by extending the Lagrangian of the SM via non-renormalizable effective interactions of (mass) dimension six or higher, i.e. by considering the following Effective Lagrangian,

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \sum_i \frac{C_i}{\Lambda^2} O_i^{d=6} + \ldots \tag{7}$$

where the dots stay for operators of dimension higher than six. The coefficients of the dimension-six operators have been written factoring out explicitly their dimensionality (m^{-2}), where Λ denotes the mass scale of new degrees of freedom belonging to a not yet known extension of the SM. The dimension-six operators that extend the SM Lagrangian have been systematically classified and a convenient basis (made of 59 operators) has been introduced in Ref. [3].

To fix the notation, the SM Lagrangian is assumed to be,

$$\begin{aligned}\mathcal{L}_{\text{SM}} = &-\frac{1}{4} G^A_{\mu\nu} G^{A\mu\nu} - \frac{1}{4} W^I_{\mu\nu} W^{I\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + (D_\mu H)^\dagger (D^\mu H) + \mu^2 H^\dagger H - \lambda (H^\dagger H)^2 \\ &+ i\big(\overline{L} D_\mu \gamma^\mu L + \overline{e}_R D_\mu \gamma^\mu e_R + \overline{Q} D_\mu \gamma^\mu Q + \overline{u}_R D_\mu \gamma^\mu u_R + \overline{d}_R D_\mu \gamma^\mu d_R\big) \\ &- \big(\overline{L}\, Y_e\, e_R H + \overline{Q}\, Y_u\, u_R \widetilde{H} + \overline{Q}\, Y_d\, d_R H + \text{h.c.}\big)\,,\end{aligned} \tag{8}$$

where $\widetilde{H} = i\sigma_2 H^*$, and the gauge-field strengths are defined as

$$G^A_{\mu\nu} = \partial_\mu G^A_\nu - \partial_\nu G^A_\mu + g_s f^{ABC} G^B_\mu G^C_\nu \,, \tag{9}$$
$$W^I_{\mu\nu} = \partial_\mu W^I_\nu - \partial_\nu W^I_\mu + g\varepsilon^{IJK} W^J_\mu W^K_\nu \,, \tag{10}$$
$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \,, \tag{11}$$

where f^{ABC} and ε^{IJK} are the structure constants of $SU(3)$ and $SU(2)$: $[T^A, T^B] = if^{ABC}T^C$ with $f^{ABC} = i(T^A)^{BC}$ for $SU(3)$, and $[\sigma^I/2, \sigma^J/2] = i\varepsilon^{JIK}\sigma^K/2$ with ε^{JIK} being the totally antisymmetric tensor. We define the covariant derivative as

$$D_\mu q_L = (\partial_\mu - ig_s T^A G^A_\mu - igS^I W^I_\mu - ig' Y_q B_\mu) q_L \tag{12}$$

for the left-handed quark, where $T^A = \lambda^A/2$, $S^I = \sigma^I/2$, and the hypercharge is normalized as $Y_q = 1/6$.

2.a) Make sure you understand how the basis of operators in Tables 2 and 3 of Ref. [3] has been built requiring just that the new interactions (operators) are expressed in terms of fields of the SM and respect the gauge symmetry of the SM. In this problem we will focus just on a subset of the operators of Table 2 that we will relabel as follows:

$$\begin{aligned}
O_{HG} &= (H^\dagger H) G^A_{\mu\nu} G^{A,\mu\nu} \,, \\
O_{HW} &= (H^\dagger H) W^I_{\mu\nu} W^{I,\mu\nu} \,, \\
O_{HB} &= (H^\dagger H) B_{\mu\nu} B^{\mu\nu} \,, \\
O_{HWB} &= (H^\dagger \tau^I H) W^I_{\mu\nu} B^{\mu\nu} \,, \\
O_{HD} &= (H^\dagger D^\mu H)^* (H^\dagger D_\mu H) \,, \\
O_{H\Box} &= (H^\dagger H) \Box (H^\dagger H) \,, \\
O_H &= (H^\dagger H)^3 \,.
\end{aligned} \tag{13}$$

2.b) Notice that the O_H operator modifies the Higgs potential and generates a shift in the Higgs VEV. Calculate it and explain how this can be reabsorbed in a redefinition of the physical Higgs field. Is this effect measurable? It is convenient (here and in the following) to use unitary gauge and write the Higgs doublet H as

$$H = \left(0, \frac{v+h}{\sqrt{2}}\right)^T \,,$$

where h is the physical Higgs field.

2.c) Show that the operators O_{HG}, O_{HW}, O_{HB}, O_{HD}, and $O_{H\Box}$ modify the kinetic terms of the gauge and Higgs bosons as follows,

$$\begin{aligned}
\mathcal{L}_{\text{kin}} = &-\frac{1}{4}\left(1 - 2\widehat{C}_{HG}\right) \tilde{G}^A_{\mu\nu} \tilde{G}^{A\mu\nu} - \frac{1}{4}\left(1 - 2\widehat{C}_{HW}\right) \tilde{W}^I_{\mu\nu} \tilde{W}^{I\mu\nu} \\
&- \frac{1}{4}\left(1 - 2\widehat{C}_{HB}\right) \tilde{B}_{\mu\nu} \tilde{B}^{\mu\nu} + \frac{1}{2}\left(1 + \frac{1}{2}\widehat{C}_{HD} - 2\widehat{C}_{H\Box}\right) (\partial^\mu \tilde{h})(\partial_\mu \tilde{h}) \,.
\end{aligned} \tag{14}$$

where the *tilded* objects in Eq. (14) denote (here and in the following) pure SM fields and couplings (i.e. elements of $\mathcal{L}_{\text{SM}}$ in the absence of NP effects), and, for simplicity, a factor of

v^2/Λ^2 has been reabsorbed into the coefficients $\widehat{C}_i \equiv \frac{v^2}{\Lambda^2} C_i$ (notice that there is no difference between v and $\tilde{v}$ beyond the linear order in the NP couplings). Rescaling the gauge fields and the corresponding gauge couplings simultaneously as,

$$G^A_\mu = \tilde{G}^A_\mu \left(1 - \widehat{C}_{HG}\right), \qquad g_s = \tilde{g}_s \left(1 + \widehat{C}_{HG}\right) \tag{15}$$

$$W^I_\mu = \tilde{W}^I_\mu \left(1 - \widehat{C}_{HW}\right), \qquad g = \tilde{g} \left(1 + \widehat{C}_{HW}\right), \tag{16}$$

$$B_\mu = \tilde{B}_\mu \left(1 - \widehat{C}_{HB}\right), \qquad g' = \tilde{g}' \left(1 + \widehat{C}_{HB}\right), \tag{17}$$

one obtains the canonically normalized kinetic term, while the covariant derivative has the same form as the original one. Hence the above shifts in the kinetic terms of the gauge bosons are not observables. On the other hand, show that the shift in the Higgs kinetic term leads to a redefinition of the Higgs field as

$$\tilde{h} = h(1+\delta_h) = h\left(1 - \frac{1}{4}\widehat{C}_{HD} + \widehat{C}_{H\Box}\right), \tag{18}$$

and leads to visible effects in the Higgs mass

$$m_h^2 = 2\lambda v^2 \left(1 - \frac{3}{2\lambda}\widehat{C}_H - \frac{1}{2}\widehat{C}_{HD} + 2\widehat{C}_{H\Box}\right), \tag{19}$$

which now depends on the quartic coupling λ.

2.d) Consider the effective Lagrangian for the HVV interactions in the mass basis and using unitary gauge,

$$\begin{aligned}
\mathcal{L}_{\text{eff}} = {} & g_{hgg} G^A_{\mu\nu} G^{A\mu\nu} h + g^{(1)}_{hWW} W^{\mu\nu} W^\dagger_{\mu\nu} h + \left(g^{(2)}_{hWW} W^{+\nu} \partial^\mu W^\dagger_{\mu\nu} h + \text{h.c.}\right) + g^{(3)}_{hWW} W^+_\mu W^{-\mu} h \\
& + g^{(1)}_{hZZ} Z_{\mu\nu} Z^{\mu\nu} h + g^{(2)}_{hZZ} Z_\nu \partial_\mu Z^{\mu\nu} h + g^{(3)}_{hZZ} Z_\mu Z^\mu h \\
& + g^{(1)}_{hZA} Z_{\mu\nu} F^{\mu\nu} h + g^{(2)}_{hZA} Z_\nu \partial_\mu F^{\mu\nu} h + g_{hAA} F_{\mu\nu} F^{\mu\nu} h \,,
\end{aligned} \tag{20}$$

where $W_{\mu\nu}$, $W^\dagger_{\mu\nu}$, $Z_{\mu\nu}$, and $F_{\mu\nu}$ are the gauge-boson strength tensors in the mass basis,

$$W_{\mu\nu} = \partial_\mu W^+_\nu - \partial_\nu W^+_\mu \,, \tag{21}$$

$$W^\dagger_{\mu\nu} = \partial_\mu W^-_\nu - \partial_\nu W^-_\mu \,, \tag{22}$$

$$Z_{\mu\nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu \,, \tag{23}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \,, \tag{24}$$

and show that the direct NP contributions to the HVV couplings read,

$$\begin{aligned}
\mathcal{L}^{\text{direct}}_{hVV} = {} & \frac{1}{v}\widehat{C}_{HG}\, G^A_{\mu\nu} G^{A\mu\nu} h + \frac{2}{v}\widehat{C}_{HW} W^{\mu\nu} W^\dagger_{\mu\nu} h + \frac{1}{v}\left[c_W^2 \widehat{C}_{HW} + s_W^2 \widehat{C}_{HB} + s_W c_W \widehat{C}_{HWB}\right] Z_{\mu\nu} Z^{\mu\nu} h \\
& + \frac{M_Z^2}{v}\widehat{C}_{HD}\, Z_\mu Z^\mu h + \frac{1}{v}\left[2 s_W c_W \left(\widehat{C}_{HW} - \widehat{C}_{HB}\right) - (c_W^2 - s_W^2)\widehat{C}_{HWB}\right] Z_{\mu\nu} F^{\mu\nu} h \\
& + \frac{1}{v}\left(s_W^2 \widehat{C}_{HW} + c_W^2 \widehat{C}_{HB} - s_W c_W \widehat{C}_{HWB}\right) F_{\mu\nu} F^{\mu\nu} h \\
= {} & \frac{1}{v}\widehat{C}_{HG}\, G^A_{\mu\nu} G^{A\mu\nu} h + \frac{2}{v}\widehat{C}_{HW} W^{\mu\nu} W^\dagger_{\mu\nu} h + \frac{1}{v}\delta_{ZZ} Z_{\mu\nu} Z^{\mu\nu} h + \frac{M_Z^2}{v}\widehat{C}_{HD}\, Z_\mu Z^\mu h \\
& + \frac{1}{v}\delta_{AZ} Z_{\mu\nu} F^{\mu\nu} h + \frac{1}{v}\delta_{AA} F_{\mu\nu} F^{\mu\nu} h \,,
\end{aligned} \tag{25}$$

where δ_{ZZ}, δ_{AA}, δ_{AZ} and δ_h are defined by

$$\delta_{ZZ} = c_W^2 \widehat{C}_{HW} + s_W^2 \widehat{C}_{HB} + s_W c_W \widehat{C}_{HWB}\,, \tag{26}$$

$$\delta_{AA} = s_W^2 \widehat{C}_{HW} + c_W^2 \widehat{C}_{HB} - s_W c_W \widehat{C}_{HWB}\,, \tag{27}$$

$$\delta_{AZ} = 2 s_W c_W \left(\widehat{C}_{HW} - \widehat{C}_{HB}\right) - (c_W^2 - s_W^2)\widehat{C}_{HWB}\,, \tag{28}$$

$$\delta_h = -\frac{1}{4}\widehat{C}_{HD} + \widehat{C}_{H\Box}\,. \tag{29}$$

For completeness, there is also an indirect contributions arising just from the effect of NP on the SM couplings and fields,

$$\begin{aligned}\mathcal{L}_{hVV}^{\text{indirect}} &= \frac{\tilde{g}^2 v}{2}\tilde{W}_\mu^\dagger \tilde{W}^\mu \tilde{h} + \frac{(\tilde{g}^2 + \tilde{g}'^2)v}{4}\tilde{Z}_\mu \tilde{Z}^\mu \tilde{h} \\ &= 2\left(\sqrt{2}\,G_F\right)^{1/2} c_W^2 M_Z^2 \left[1 - \frac{1}{2(c_W^2 - s_W^2)}\left(4 s_W c_W \widehat{C}_{HWB} + c_W^2 \widehat{C}_{HD} + \delta_{G_F}\right) + \delta_h\right] W_\mu^\dagger W^\mu h \\ &\quad + \left(\sqrt{2}\,G_F\right)^{1/2} M_Z^2 \left(1 - \frac{1}{2}\widehat{C}_{HD} + \delta_h - \frac{1}{2}\delta_{G_F}\right) Z_\mu Z^\mu h\,, \end{aligned} \tag{30}$$

where we can recognize for instance the effect of the shift in the Higgs field (δ_h). The other terms come from the redefinition of the weak couplings in terms of shifts in M_W, M_Z, s_W, and G_F.

2.e) Using Eq.s (25)-(30) build a table where you list for each coupling in Eq. (20): the SM tree-level value, the direct NP effect, and the indirect NP effect. What are the new Feynman rules for the hZZ, hWW, $h\gamma\gamma$, and hgg vertices?

Can you explain why Higgs data already strongly constrain C_{HWB}, C_{HW}, C_{HB}, and C_{HG}?

References

[1] J. D. Wells, arXiv:0909.4541 [hep-ph].

[2] L. Reina, arXiv:1208.5504 [hep-ph].

[3] B. Grzadkowski, M. Iskrzynski, M. Misiak and J. Rosiek, JHEP **1010**, 085 (2010) [arXiv:1008.4884 [hep-ph]].