Name:________________________________

PHY 3221 : Intermediate Mechanics, Spring 2002

March 8^{th}, 2002

Midterm Exam # 2

Note. There are three problems. *All problems are marked out of 10, and are equally weighted.* Take a few minutes to read them and decide where you want to start from.

You may use any technique you like to solve the following problems. But always remember to write full work for what you do. This will help your grade in case of incomplete or wrong answers. Also, no credit will be given for an answer, even if correct, if you give no justification for it. I will take anything you write at face value, so be careful with vectors, equal signs, etc.

Write your final answers on the sheets provided. You may separate them as long as you put your name on each of them. We will staple them when you hand them in. Ask if you need extra sheets, they will be provided. Remember to put your name on each of them and add them after the problem they refer to.

Good luck!

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Problem 1

A small lead ball of mass m is constrained on a table by means of four light springs, such that the potential energy of the lead ball can be written as the potential energy of an anisotropic 2-dimensional harmonic oscillator:

$$U(x,y) = \frac{1}{2}k\left(4x^2 + 16y^2\right)$$

where k is a real positive constant.

(a) Write the equations of motion of the lead ball (both x- and y-components).

(b) Knowing that at time $t=0$ the lead ball is displaced at a generic point of coordinates $(a,0)$ and released with an initial velocity of components $(0,v_0)$, find the explicit expression of $x(t)$ and $y(t)$ at a later time $t>0$.

(c) Show that the motion of the lead ball is periodic and find after how long the lead ball returns to the initial position $(a,0)$ moving with the same initial velocity $(0,v_0)$.

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Problem 2

The equation of motion of a mass m moving in a viscous fluid is:

$$\ddot{x} + 4\dot{x} + 8x = 0$$

(a) Is the mass behaving as an underdamped, critically damped or overdamped oscillator? Explain why.

(b) Write the solution of the equation of motion, $x(t)$, knowing that $x(t=0)=x_0$ and $\dot{x}(t=0)=0$. Define carefully all the symbols you introduce in solving this problem.

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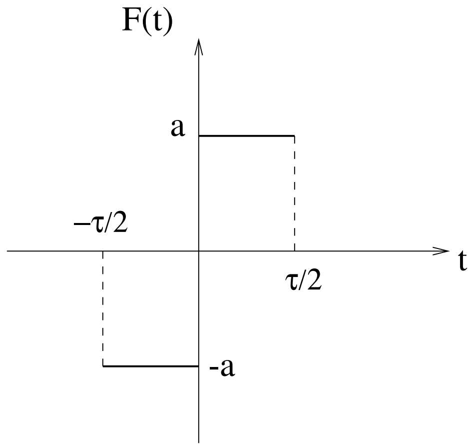
Problem 3

An underdamped oscillator is forced by an external periodic force of the form (τ indicates the period of the function and a is a positive real number):

$$F(t) = \begin{cases} -a & -\frac{\tau}{2} < t < 0 \\ a & 0 < t < \frac{\tau}{2} \end{cases}$$



as illustrated in the figure.

(a) Write $F(t)$ as a Fourier series, and find the coefficients of the series explicitly.

(b) Explain why it is important to be able to write a generic periodic external force acting on a damped oscillator as the superposition of sinusoidal terms of the form $\sin(n\omega t)$ and $\cos(n\omega t)$, where ω is the frequency of the given periodic force and n is the summation index in the Fourier expansion you should have derived in point **(a)**.

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