PHY 3221 : Intermediate Mechanics, Spring 2002

January 11^{th}, 2002

Assignment # 1

(due Friday January 18^{th}, 2002, at the beginning of class)

1. Problem 1.9 of Marion and Thornton's book.

2. Consider a cube whose edges are each of unit length. One corner coincides with the origin of an $Oxyz$ Cartesian coordinate system. Three of the cube's edges extend from the origin along the positive direction of each coordinate axis. Find the vector that begins at the origin and extends:

(a) along a major diagonal of the cube;

(b) along the diagonal of the lower face of the cube.

(c) Calling these vectors **A** and **B**, find **C**=**A**×**B**.

(d) Find the angle between **A** and **B**.

3. The motion of a projectile is described by the following equation

$$\mathbf{r}(t) = \mathbf{i}\, bt + \mathbf{j}\left(ct - \frac{gt^2}{2}\right) + \mathbf{k}\, 0$$

(a) In spite of the three dimensional notation, this a planar motion (in the (x, y) plane). Can you see why?

(b) Calculate $\mathbf{v}(t)$ and $\mathbf{a}(t)$.

(c) Describe the motion in the x-direction. What is b?

(d) Describe the motion in the y-direction. What is c? What is g?

4. A buzzing fly moves in a helical path given by the equation

$$\mathbf{r}(t) = \mathbf{i}\, b\sin(\omega t) + \mathbf{j}\, b\cos(\omega t) + \mathbf{k}\, ct^2$$

(a) Show that the magnitude of the acceleration of the fly is constant, provided b, c and ω are constant.

(b) Can you figure out and describe what the trajectory of the fly looks like?

(c) Check your intuition with *Maple*, using the **spacecurve** command. Take b=c=1 and ω=2 and try:

```
> with(plots);
> b:=1; c:=1; omega:=2;
>spacecurve([b*sin(omega*t),b*cos(omega*t),c*t^2],t=0..4*Pi,numpoints=100);
```

5. Read and practice your *Maple* tutorial. Then try the following commands of *Maple* and explain what's wrong with them (**read and practice the first tutorial!**) Correct them and write down the answers you get.

   (a) tan(pi/4);

   (b) Evalf(Pi);

   (c) plot(sin(a*t),t=0..2*Pi);

6. **For graduates** (bonus for undergraduates)
   Show that:

$$\sum_{k=1}^{3} \epsilon_{ijk}\epsilon_{lmk} = \delta_{il}\delta_{jm} - \delta_{im}\delta_{jl}$$

   and evaluate the sum in the case in which: **(i)** $i = j$ or $l = m$, **(ii)** $i = l$ and $j = m$, **(iii)** $i = m$ and $j = l$. Use this result to prove that:

$$\boldsymbol{A} \times (\boldsymbol{B} \times \boldsymbol{C}) = (\boldsymbol{A} \cdot \boldsymbol{C})\, \boldsymbol{B} - (\boldsymbol{A} \cdot \boldsymbol{B})\, \boldsymbol{C}$$

7. **For graduates** (bonus for undergraduates)
   Show that:

$$\text{(a)} \qquad \nabla r = \frac{\mathbf{r}}{r} = \mathbf{e_r}$$

$$\text{(b)} \qquad \nabla\left(\frac{1}{r}\right) = -\frac{\mathbf{r}}{r^3} = -\frac{1}{r^2}\mathbf{e_r}$$

   where r is the magnitude of the position vector $\mathbf{r} = x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k}$ and $\mathbf{e_r}$ is the unit vector in the direction of $\mathbf{r}$.

   Loading the **plots** package in *Maple*, plot the previous two gradients in D=2 (dimensions) using the function **gradplot** and in D=3 (dimensions) using the function **gradplot3d**. A *gradient plot* is a pictorial way to represent a gradient function (like the ones you have in **(a)** and **(b)** above. Since a gradient function is a *vector function*, it associates a vector to each point in space. A gradient plot will pick a sample of points in the region of space you want and draw an arrow for each of them, directed as and proportional to the vector function in that point.

   I suggest you use the following options to get readable plots:

   (i) For the **D=2 case**:
   gradplot$((x^2 + y^2)^{(\pm 1/2)}, x = -1..1, y = -1..1, \text{grid} = [10, 10]);$

   (i) For the **D=3 case**:
   gradplot3d$((x^2 + y^2 + z^2)^{(\pm 1/2)}, x = -1..1, y = -1..1, z = -1..1, \text{grid} = [5, 5, 5]);$

   Note that the ranges for (x, y, z) can be changed as you like. If you then click on your plot, more optional commands will appear in the top bar of your window. Use them to move the coordinate axes as you like.

   Print out your result and explain what do you understand from these plots. Do you see any difference between them? For instance:

   i) Are the arrows directed the same way in case **(a)** and **(b)**? Why?

   ii) Are the length of the arrows always the same in each plot? If yes, why? If no, why?