PHY 3221 : Intermediate Mechanics, Spring 2002

March 22^{nd}, 2002
Assignment # 10
(due Friday March 29^{th}, 2002, at the beginning of class)

1. Write the unit vectors $\mathbf{\hat{e}_r}$ and $\mathbf{\hat{e}}_\theta$ in terms of the cartesian unit vectors $\hat{\imath}$ and $\hat{\jmath}$ and the polar coordinates r and θ. Use this result to show that

$$\frac{d}{dt}\mathbf{\hat{e}}_r = \dot{\theta}\mathbf{\hat{e}}_\theta \qquad \frac{d}{dt}\mathbf{\hat{e}}_\theta = -\dot{\theta}\mathbf{\hat{e}}_r$$

2. A particle moves with constant speed v around a semi-circle of radius b. Find the velocity in polar coordinates

(a) using an origin at the center of the semi-circle;

(b) using an origin lying on the left edge of the semi-circle.

3. A particle is moving in such a way that the radial distance decreases at a constant rate, $r = b - ct$, while the angular speed increases at a constant rate, $\dot{\theta} = kt$. Find the speed and the acceleration as functions of time.

4. A block of mass m is attached to a massless spring with spring constant k. The other extremum of the spring is attached to a fixed pivot on a frictionless horizontal table. Imagine to slightly displace the block from its equilibrium position in an arbitrary direction in the plane of the table (give your answer in plane polar coordinates (r, θ) with origin at the pivot).

(a) What is the force $\mathbf{F}$ on the block and what potential does the block move in?

(b) Show by direct calculation that the magnitude of the angular momentum of the block is $l = mr^2\dot{\theta}$. Is the angular momentum conserved? Why?

(c) Find the total energy $E = T + U$ in terms of the radial coordinates $(r, \dot{r})$, and so find the effective potential for the radial motion $V_{eff}(r)$. Plot $V_{eff}(r)$ and discuss the kind of motion you may expect for this system.

(d) Show that the system has a stable circular orbit.

(e) If $r = r_0$ is the values of the radius of the stable circular orbit of the system, show that for a small radial displacement about r_0 (i.e. for $r_0 \to r_0 + x$, with x small) the radial motion of the system is simple harmonic. Find the frequency of the harmonic oscillations.

5. Problem 8.14 of Marion and Thornton's book.

6. **For graduates** (bonus for undergraduates).
A particle of mass m and position vector $\mathbf{r}$ is subject to two forces: a central force $\mathbf{f_1} = F(r)\mathbf{\hat{e}_r}$, and a frictional force $\mathbf{f_2} = -b\mathbf{v}$, with $b > 0$.

(a) Is the motion of the particle two or three dimensional? Why?

(b) If the particle initially has angular momentum $\mathbf{L_0}$ about the origin, find the angular momentum at all subsequent times using $\tau = \dot{\mathbf{L}}$.

7. For graduates (bonus for undergraduates).
Problem 8.6 of Marion and Thornton's book.