PHY 3221 : Intermediate Mechanics, Spring 2002

February 8^{th}, 2002

Assignment # 5

(due Friday February 15^{th}, 2002, at the beginning of class)

1. The potential energy for a vibrating diatomic molecule can be approximated by the following function:

$$U(x) = U_0 \left[1 - e^{-(x-x_0)/\delta}\right]^2 - U_0$$

where x is the separation distance between the two constituent atoms, and x_0, U_0 and δ are parameters chosen to describe the observed behavior of a particular molecule (the meaning of these parameters becomes clearer in solving the problem).

(a) Does the system admit any equilibrium point? If yes, is the equilibrium stable or unstable? Justify your answer showing full work. What is the meaning of x_0 and U_0 in the parameterization of $U(x)$?

(b) Draw a qualitative sketch of the function $U(x)$ and describe the relative motion of the two atoms (i.e. the vibrations of the molecule) when the energy of the system is: $E = -U_0$, $-U_0 < E < 0$, and $E \geq 0$.

(c) What is the force that each atom exerts on the other?

(d) Show that for separation distances x close to x_0 the potential energy is parabolic and the force is linear and always directed towards the equilibrium point (restoring force). (Remember that the Taylor expansion of the exponential function about a given point x_0 is: $\exp(x - x_0) = 1 + (x - x_0) + (x - x_0)^2/2 + O((x - x_0)^3)$).

2. Problem 3.1 of Marion and Thornton's book.

3. Problem 3.3 of Marion and Thornton's book.

4. When a light spring supports a block of mass m in a vertical position, the spring is found to be stretched by an amount D_1 over its unstreched length. If the block is furthermore pulled downward a distance D_2 from the equilibrium position and released at time $t=0$, find:

(a) the resulting motion $x(t)$;

(b) the velocity of the block when it passes back upward through the equilibrium position;

(c) the acceleration of the block at the top of its oscillatory motion.

5. A particle of mass m moves in two dimensions under the following potential energy function:

$$V(x, y) = \frac{1}{2}\, k\, (x^2 + 4\, y^2) \ ,$$

(a) Find the resulting motion, given the initial condition at $t=0$: $x=a$, $y=0$, $\dot{x}=0$, $\dot{y}=v_0$.

(b) Draw the trajectory using *Maple* (you can easily do this using the **plot** command in parametric mode). Is the motion periodic? Could you predict this result from the expression of $x(t)$ and $y(t)$?

6. **For graduates** (bonus for undergraduates)
Problem 3.7 of Marion and Thornton's book.