PHY 3221 : Intermediate Mechanics, Spring 2003

April 11^{th} , 2003 Assignment # 12 (due Friday April 18^{th} , 2003, at the beginning of class)

- 1. Calculate the moment of inertia of a uniform disk of radius R and mass M with respect to an axis through the center and perpendicular to the plane of the disk. Repeat the calculation for an axis through the center and in the plane of the disk.
- 2. Calculate the moment of inertia of a uniform hollow cylinder of mass M, radius R, and length L with respect to the central longitudinal axis of the cylinder. Repeat the calculation for a uniform solid cylinder.
- 3. Two balls of mass m are attached at the ends of a thin rod of negligible mass and length l. The rod is free to rotate without friction about a horizontal axis through the center. Initially the rod is at rest. A putty wad of mass M drops vertically with speed v onto the ball on the right hand side and sticks to it.
 - **3.a)** What is the angular momentum of the entire system (including balls, rod, and putty wad) about the axis before the putty wad drops onto the ball? Assume the positive direction of the z axis to be the one pointing into the paper.
 - **3.b)** What is the total moment of inertia of the balls-rod-putty wad system about the axis after the putty wad sticks to the ball?
 - **3.c)** What is the angular speed of the system just after the putty wad hits the ball?
 - **3.d)** Bonus: Repeat the problem for the case of a massive rod of mass \overline{m} .
- 4. Consider a ball of mass m striking a vertically held cricket bat of mass M (to be considered approximately as a uniform rod of length L). The ball impacts at a point x with horizontal velocity \vec{u} . The bat is held lightly so that at the instant of the ball's impact the bat may be considered free. After the collision the ball moves with velocity \vec{v} .



4.a) Show that the moment of inertia of the bat about an axis perpendicular to the bat's length through the center of mass is $ML^2/12$.

- **4.b)** If \vec{V} and ω are respectively the velocity of the center of mass and the magnitude of the angular velocity of the bat after the collision, write down the equations of conservation of linear and angular momentum of the system.
- 4.c) What is the magnitude of the angular velocity of the bat, ω , after the collision?
- **4.d)** Bonus: By noting that the angular motion induces a linear velocity of $-\omega L/2$ at the top of the bat, show that the net velocity of the top of the bat is zero when x = 2L/3.