PHY 3221 : Intermediate Mechanics, Spring 2003

January 17^{th}, 2003

Assignment # 2

(due Friday January 25^{th}, 2003, at the beginning of class)

1. Problem 2.14 of Marion and Thornton's book.

2. Problem 2.32 of Marion and Thornton's book.

3. Problem 2.9 of Marion and Thornton's book. Assume that the resisting force is of the form $F_r = -mkv$. After having completed points **(a)** and **(b)** in Problem 2.9:

(c) Compare the times found in **(a)** and **(b)** analytically, and explain why the time in **(b)** is always smaller than the time in **(a)**.

(d) Take the limits $k \to 0$ and $k \to \infty$ in the expression for the time obtained in **(b)** and explain your results.

4. A gun is fired straight up. Assuming that the air drag on the bullet varies quadratically with speed (i.e. $F_r = -mkv^2$ for a motion vertically upward or $F_r = mkv^2$ for a motion vertically downward, when you measure the vertical coordinate positive upward), show that the speed varies with height according to the equations:

$$v^2 = (v_t^2 + v_0^2)\, e^{-2kx} - v_t^2 \quad \text{(upward motion)} \ ,$$

$$v^2 = v_t^2 - v_t^2 e^{2kx} \quad \text{(downward motion)} \ ,$$

where x is the vertical displacement, g is the gravitational acceleration, v_0 is the initial speed with which the bullet is fired, and $v_t = \sqrt{g/k}$ is the terminal speed of the bullet. (*Hints*: in order to find the speed as a function of the position, i.e. $v = v(x)$, observe that the equation of motion can also be written as:

$$m\,a = m\frac{dv}{dt} = m\frac{dv}{dx}\frac{dx}{dt} = m\,v\,\frac{dv}{dx} = F_{net} \ ,$$

where F_{net} is the sum of all the forces acting on the bullet. Moreover, observe that the equation of motion is slightly different for the upward and the downward motions: since the retarding force is proportional to the square of the velocity, you need to carefully switch the sign of F_r when the bullet moves upward or downward.)

5. Use the result is **Problem 4** to show that, when the bullet hits the ground on its return, the speed v_f will be given by

$$v_f = \frac{v_t v_0}{\sqrt{v_t^2 + v_0^2}} \ ,$$

where v_0 and v_t have been defined in **Problem 4**.