PHY 3221 : Intermediate Mechanics, Spring 2003

January 31^{st}, 2003

Assignment # 4

(due Friday February 7^{th}, 2003, at the beginning of class)

1. Problem 3.1 of Marion and Thornton's book.

2. When a light spring supports a block of mass m in a vertical position, the spring is found to be stretched by an amount D_1 over its unstreched length. If the block is furthermore pulled downward a distance D_2 from the equilibrium position and released at time $t=0$, find:

(a) the resulting motion $x(t)$;

(b) the velocity of the block when it passes back upward through the equilibrium position;

(c) the acceleration of the block at the top of its oscillatory motion.

3. A particle of mass m moves in two dimensions under the following potential energy function:

$$V(x,y) = \frac{1}{2}\,k\,(x^2 + 4\,y^2) \ ,$$

(a) Find the resulting motion, given the initial condition at $t=0$: $x=a$, $y=0$, $\dot{x}=0$, $\dot{y}=v_0$.

(b) Draw the trajectory using *Maple* (you can easily do this using the **plot** command in parametric mode). Is the motion periodic? Could you predict this result from the expression of $x(t)$ and $y(t)$?

4. Problem 3.2 of Marion and Thornton's book.

5. The total mechanical energy of a damped oscillator decreases with time. For the case of an underdamped oscillator:

(a) Derive the expression for the energy ($E=T+U$) and the energy loss (dE/dt) and show that dE/dt is proportional to the square of the velocity (*Hint*: using the equation of motion of the damped oscillator can be very helpful for the last part).

(b) Using *Maple*, show $U(t)$, $T(t)$, and $E(t)$ on the same plot, and dE/dt on a separate plot. Make sure that you can reproduce Fig. 3-7 in your book. Explain why you can see from your plots that $dE/dt \propto v^2$.

6. For graduates (bonus for undergraduates)
Problem 3.7 of Marion and Thornton's book.