PHY 3221 : Intermediate Mechanics, Spring 2003

February 14^{th}, 2003

Assignment # 6

(due Friday February 21^{st}, 2003, at the beginning of class)

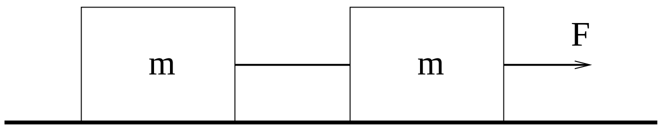
1. Consider the force $F(t)$ with periodic triangular shape discussed in class:

$$F(t) = F_0 \left\{ \begin{array}{ll} -\frac{4}{\tau}\left(t - \frac{\tau}{4}\right) & 0 \le t \le \frac{\tau}{2} \\ \frac{4}{\tau}\left(t + \frac{\tau}{4}\right) & -\frac{\tau}{2} \le t \le 0 \end{array} \right.$$

Obtain its Fourier expansion. Plot the function and the sums of the first two, three, four and five terms. Discuss your results.

2. Problem 3.32 of Marion and Thornton's book.

3. Two blocks of equal mass m are attached to each other by an inextensible string and placed on a horizontal table. A constant horizontal force $\mathbf{F}$ is applied to one of the two blocks (see figure). The coefficient of static and kinetic friction between the blocks and the table are μ_s and μ_k respectively.



(a) Find the minimum force necessary for the blocks to start moving.

(b) If a force of magnitude larger than the value you obtained in point **(a)** is applied to one of the two blocks, find the acceleration of the two blocks.

(c) Under the same assumptions as in point **(b)**, find the tension in the string linking the two blocks.

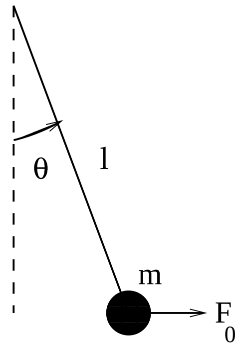
4. The equation of motion of a mass m moving in a viscous fluid is:

$$\ddot{x} + 4\dot{x} + 8x = 0$$

(a) Is the mass behaving as an underdamped, critically damped or overdamped oscillator? Explain why.

(b) Write the solution of the equation of motion, $x(t)$, knowing that $x(t=0) = x_0$ and $\dot{x}(t=0) = 0$. Define carefully all the symbols you introduce in solving this problem.

5. A pendulum consisting of a bob of mass m attached to a string of length l is subject to a constant horizontal external force F_0.



(a) Find the equation of motion of the pendulum assuming that its angular displacement is small and knowing that at $t = 0$ $\theta(0) = 0$ and $\dot{\theta}(0) = v_0$. Explain your result.

(b) Imagine now that the same system is surrounded by a medium which retards the bob's motion with a resistive force proportional to its speed, of magnitude $F_r = m\sqrt{g/l}(l\dot{\theta})$. Write the equation of motion for the pendulum in this new environment, and solve it for the same initial conditions given in point **(a)**. Which kind of damped motion have you obtained?

(c) After how long the amplitude of the motion is reduced to one half of its initial value?