PHY 4241/5227 : Advanced Dynamics, Spring 2007

January 12^{th}, 2007
Assignment # 1
(due Friday January 19^{th}, 2007, at the beginning of class)

Note: For each problem of this homework, even if not explicitly requested, I would like you to justify your solution step by step. This will help you understanding the Lagrangian formalism without applying it in a pedestrian way. You are welcome to solve any problem using software packages like *Maple* or *Mathematica*. However, also in this case, you will have to explain your procedure.

1. Consider a particle of mass $m=1$ moving from $x_1=0$ at time $t_1=0$ to $x_2=1$ at time $t_2=\pi/2$, under the influence of a one-dimensional harmonic potential of the form:

$$U(x) = \frac{1}{2}x^2$$

(a) Using Euler-Lagrange equations of motion, obtain the time-dependent motion of the system, *i.e.* solve for $x(t)$. Compute the action for the exact path.

(b) Using the approximate linear path of the form $x(t)=a+bt$, compute the action for this path and compare it with the value obtained in part **a)**.

(c) Using the approximate quadratic path of the form $x(t)=a+bt+ct^2$, compute the action for this path and compare it with the value obtained in part **a)**.

Hint: For parts **b)** and **c)** make sure that the paths are consistent with the boundary conditions. If any constant remains undetermined, fix it by minimizing the action.

2. What generalized coordinates can be used to completely specify the motion of each of the following? Explain what the constraints are in each case and how they determine the number (and the choice) of the generalized coordinates you need case by case.

(a) A particle constrained to move on a sphere.

(b) A circular cylinder rolling down an inclined plane.

(c) A disk rolling without slipping across a horizontal plane. The plane of the disk remains vertical, but it is free to rotate about a vertical axis.

3. Consider a particle constrained to move in a plane under the action of a central force field $\mathbf{F_r}=F_r\hat{\mathbf{e}}_\mathbf{r}$.

(a) Discuss the constraint(s) and your choice of generalized coordinates.

(b) Write the Lagrangian.

(c) Derive the equations of motion and explain them. How could you tell (without having to calculate the equations of the motion) that the angular momentum of the particle is conserved: (i) from the physics of the problem and (ii) from the form of the Lagrangian?

4. Problem 7.4 of Marion and Thornton's book.

5. Problem 7.5 of Marion and Thornton's book.