PHY 4241/5227 : Advanced Dynamics, Spring 2007

February 2^{nd}, 2007

Assignment # 4

(due Friday February 9^{th}, 2007, at the beginning of class)

1. Problem 7.30 of Marion and Thornton's book, where you will learn about *Poisson brackets*.

2. A particle of mass m and electric charge q moves under the influence of a constant magnetic field $\mathbf{B}(\mathbf{r}) = B_0\hat{\mathbf{z}}$.Obtain the most general solution for the velocity $\mathbf{v}(t)$ using Newton's second law of motion in combination with the Lorentz force. That is:

$$\mathbf{F} = m\dot{\mathbf{v}} = \frac{q}{c}\mathbf{v} \times \mathbf{B} \ .$$

3. The Lagrangian for a particle of mass m and electric charge q moving under the influence of a magnetic (but not electric) field is given by:

$$L(\mathbf{r}, \dot{\mathbf{r}}) = \frac{1}{2}m\dot{\mathbf{r}}^2 + \frac{q}{c}\dot{\mathbf{r}} \cdot \mathbf{A}(\mathbf{r}) \ ,$$

where $\mathbf{A}(\mathbf{r})$ is the vector potential. Assume that the magnetic field is constant and given by $\mathbf{B}(\mathbf{r}) = B_0\hat{\mathbf{z}}$.

3.a) Show that for such a constant magnetic field the vector potential can be written in the form $\mathbf{A}(\mathbf{r}) = \frac{1}{2}(\mathbf{B}\times\mathbf{r})$. That is, show that such a vector potential satisfies: $\nabla \times \mathbf{A} = \mathbf{B}$.

3.b) Construct the Hamiltonian of the system in terms of the Cartesian coordinates and the corresponding canonical momenta of the particle.

3.c) Denoting by $\boldsymbol{\pi} = m\dot{\mathbf{r}} = m\mathbf{v}$ the *mechanical* momentum of the particle, evaluate the following Poisson brackets:

$$[\boldsymbol{\pi}_x, \boldsymbol{\pi}_y] \ , \ [\boldsymbol{\pi}_y, \boldsymbol{\pi}_z] \ , \ [\boldsymbol{\pi}_z, \boldsymbol{\pi}_x] \ .$$

3.d) By re-expressing the Hamiltonian of part **3.b)** in terms of the mechanical momentum of the particle, and using the results derived in part **3.c)**, obtain the most general solution for $\pi(t)$ by using the Poisson's equation:

$$\frac{d\boldsymbol{\pi}}{dt} = [\boldsymbol{\pi}, H] \ .$$

Interpret your results on the basis of a *conventional* (Newton's second Law plus Lorentz force) approach (see Problem 1).