PHY 4241/5227 : Advanced Dynamics, Spring 2007

March 2^{nd}, 2007

Assignment # 8

(due Friday March 16^{th}, 2007, at the beginning of class)

1. Problem 9.14 of Griffiths's book.

2. Consider an electromagnetic plane wave incident on an interface that separates two media, as we did in class. Now, however, let the electric (not the magnetic) field be perpendicular to the plane of incidence.

2.a) Write the expressions for the wave vector $\mathbf{k}$, the electric field $\mathbf{E}$, and the magnetic field $\mathbf{B}$ for the incident, reflected, and transmitted wave.

2.b) Write the four boundary conditions satisfied by the fields. Show that only two of them are independent.

2.c) By solving the above equations compute the reflection and transmission intensities. Make sure that they satisfy $R+T=1$.

2.d) Make a plot of the reflection and transmission intensities as a function of the incident angle using $n_1 = 1$, $n_2 = 1.5$, and $\mu_1 = \mu_2 = 1$. Is there a Brewster's angle for this problem? That is, is there an incident angle for which R vanishes for this configuration of fields?

3. Consider a linear medium of dielectric constant ϵ and permeability μ. Further, assume that the energy density (*i.e.*, the energy per unit volume) stored in space is given by (in Gaussian units):

$$u = \frac{1}{8\pi}(\mathbf{E}\cdot\mathbf{D} + \mathbf{B}\cdot\mathbf{H}) \ .$$

3.a) Starting from the above equation, and using Maxwell's equations, show that the energy satisfies a continuity like equation of the form:

$$\frac{\partial u}{\partial t} + \nabla\cdot\mathbf{S} = -\mathbf{J}\cdot\mathbf{E} \ .$$

What is the expression of $\mathbf{S}$ in terms of electric and magnetic fields? What is the physical significance of the $\mathbf{J}\cdot\mathbf{E}$ term.

3.b) Compute the *time-averaged* Poynting vector for the electromagnetic plane wave of wave number $\mathbf{K}$ and frequency ω. Note that the definition of the time-averaged Poynting vector is given by

$$\langle\mathbf{S}\rangle_{\rm avg} = \frac{1}{T}\int_0^T \mathbf{S}(\mathbf{x},t)dt \ \ ,(T = 2\pi/\omega) \ .$$