PHY 5667 : Quantum Field Theory A, Fall 2007

August 30^{th}, 2007
Assignment # 1
(due Thursday September 6^{th}, 2007)

1. Write down the Lagrangian of a relativistic free particle and show that, in the limit $v/c \to 0$ (where v is the magnitude of the particle velocity and c is the velocity of light) it reduces to the Lagrangian of a non-relativistic free particle modulus a constant. Explain why this does not affect the form of the non-relativistic equations of motion.

2. It is often possible to derive a field theory as the limit of a discrete system. Perhaps the simplest example is an infinite system of point masses, m, separated by springs of spring constant k and equilibrium length a. Let η_i be the displacement from equilibrium of the ith point mass. Derive the exact Lagrangian and the Lagrange equations for this system. Then consider the limit

$$m, a \to 0 \ , \ k \to \infty \ , \ \mu = m/a \ \text{and} \ Y = ka \ \text{fixed} \ .$$

Replacing η_i by a smooth function $\eta(x,t)$, show that in this limit the Lagrangian may be written in the density form

$$L = \int dx \frac{1}{2}\left[\mu\left(\frac{\partial\eta}{\partial t}\right)^2 - Y\left(\frac{\partial\eta}{\partial x}\right)^2\right]$$

and write down the corresponding (partial differential) Lagrange equations.

3. The electromagnetic field may be specified by a vector $A^\mu(\mathbf{x}, t)$, in terms of which the Lagrange density of the field is

$$\mathcal{L}(x) = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} \ ,$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \ .$$

Derive the Lagrange equations for this system, and express them in terms of the free-space field strengths $\mathbf{E} = -\partial_0\mathbf{A} - \nabla A_0$ and $\mathbf{B} = \nabla \times \mathbf{A}$. How many of Maxwell's equations does this give, and why are the others also satisfied?