PHY 5524: Statistical Mechanics

January 7^{th}, 2011
Assignment # 1
(Due Wednesday January 12^{th}, 2011)

This problem is taken from *Entropy, Order Parameters, and Complexity* by James P. Sethna and goes under the name of **Quantum dice**. It deals with discrete distributions and serves as a preview/review to Bose and Fermi statistics. Here is how it goes.

You are given several unusual "three-sided" dice which, when rolled, show either one, two, or three spots. There are three games played with these dice: *Distinguishable*, *Bosons*, and *Fermions*. In each turn in these games, the player rolls one die at a time, starting over if required by the rules, until a legal combination occurs. In *Distinguishable*, all rolls are legal. In *Bosons*, a roll is legal only if the new number is larger or equal to the preceding number. In *Fermions*, a roll is legal only if the new number is strictly larger than the preceding number.

1. **[5pt]** Notice that our dice rules are the same that govern the quantum statistics of identical particles. Can you explain why? In other words: what is the role played by the dice and the role played by the sides of the dice?

2. **[5pt]** Build a table of possibilities after rolling two dice for each of the games, i.e. what are the possible legal combinations after rolling two dice when playing *Distinguishable*, *Bosons*, and *Fermions*?

3. **[5pt]** Presume that the dice are fair (i.e. each of the three numbers of dots shows up 1/3 of the time). Among the legal *Bosons* turn rolling two dice, what is the probability $\rho(4)$ of rolling a 4. Similarly, what is $\rho(4)$ when playing *Fermions*?

4. **[5pt]** For a legal turn rolling three "three-sided" dice in *Fermions*, what is the probability $\rho(6)$ of rolling 6?

5. **[10pt]** When rolling M dice each with N sides, how many legal turns there are in *Distinguishable*, *Bosons*, *Fermions*.

6. **[10pt]** In a turn of three rolls, what is the factor by which the probability of getting triplets in *Bosons* is enhanced over that in *Distinguishable*? In a turn of M rolls, what is the enhancement factor for generating an M-tuple (all rolls having the same number of dots showing)?
   Notice that the states of the dice tend to cluster together in *Bosons*. Examples of real bosons clustering into the same state include Bose condensation and lasers.