

PHY 5524: Statistical Mechanics

January 12th, 2011

Assignment # 2

(Due Wednesday January 19th, 2011)

Problem 1

Consider the problem of an isolated particle of mass m moving freely in a one-dimensional box of size L . Such a particle satisfies the following one-dimensional Schrödinger equation:

$$-\frac{\hbar^2}{2m} \frac{d^2\varphi(x)}{dx^2} = \epsilon\varphi(x) \text{ , with } \varphi(0) = \varphi(L) = 0 \text{ .}$$

1.a) Obtain the eigenvalues and the normalized eigenvectors of the Schrödinger equation.

Now consider an isolated system of 4 *non-interacting* particles of mass m placed in such one-dimensional box. The energy of the system equals $E = 63\epsilon_0$, where

$$\epsilon_0 = \frac{\hbar^2\pi^2}{2mL^2} \text{ ,}$$

is the lowest eigenvalue of the Schrödinger equation. Find the entropy of the system for the following cases:

1.b) a system of 4 *distinguishable* spinless particles;

1.c) a system of 4 *indistinguishable* spinless bosons;

1.d) a system of 4 *indistinguishable* spin-1/2 fermions.

Problem 2

Consider a system of N localized particles moving under the influence of a quantum, one-dimensional, harmonic-oscillator potential of frequency ω . The energy of the system is given by

$$E = \frac{1}{2}N\hbar\omega + M\hbar\omega \text{ ,}$$

where M is the total number of quanta in the system. That is,

$$M = \sum_{i=1}^N n_i \text{ ,}$$

with $n_i = 0, 1, 2, \dots$ representing the number of quanta in the i_{th} harmonic oscillator.

2.a) Compute the number of microstates Γ as a function of N and M .

- 2.b)** Using Sterling's approximation, compute the entropy of the system as a function of N and M .
- 2.c)** Compute the temperature T of the system as a function of N and M . Are there any values of N and M for which the temperature T becomes negative?
- 2.d)** Compute the specific heat C_v of the system as a function of N and T . Note that in order to do so, you will need to express the total energy of the system as a function of N and T .
- 2.e)** Compute the low- ($k_B T \ll \hbar\omega$) and high-temperature ($k_B T \gg \hbar\omega$) limits of the specific heat, and make a simple plot of its behavior as a function of temperature.

Problem 3

Problem 1.4 of Pathria's book.