PHY 5524: Statistical Mechanics

February 23^{th}, 2011
Assignment # 7
(Due Wednesday March 2^{nd}, 2011)

Start reading sections 5.4, 6.1-6.2 of Pathria's book (you will need only parts of them) and think of how to approach the following problems. We will discuss both problems in class on Monday, February 28.

Problem 1

Two non-interacting particles inhabit a potential well such that the orbital motion of each particle gives rise to an energy spectrum $E(n) = n\epsilon$, the n^{th} energy level having a degeneracy $g(n) = 2n + 1$. Consider spin-0 particles obeying Bose-Einstein statistics, spin-$\frac{1}{2}$ particles obeying Fermi-Dirac statistics, and spin-s particles obeying Maxwell-Boltzmann statistics. Assuming that the energy is spin-independent,

1.a) find the microcanonical partition function of the system when the total energy has the fixed value $E = N\epsilon$,

1.b) find the canonical partition function of the system when it is in contact with a heat reservoir at fixed temperature T, and

1.c) investigate the relationship between the partition functions obtained for the three types of statistics.

Problem 2

Consider a system of non-interacting indistinguishable particles whose states can be specified in the following way.

(i) There are single particle states, labelled by an index i, with energy ϵ_i, which will be degenerate (ϵ_i may have the same value for several values of i) for particles with non-zero spin.

(ii) Each multiparticle state corresponds to a set of occupation numbers $\{n_i\}$, where n_i counts the number of particles occupying the i^{th} single-particle state and has values from 0 to M. Each distinct set of occupation numbers corresponds to a distinct state.

Use the grand-canonical ensemble, and consider separately the cases in which the system obeys Bose-Einstein, Fermi-Dirac, or Maxwell-Boltzmann statistics.

2.a) Obtain the probability distribution(s) $P_i(n_i, T, \mu)$ for finding n_i particles in a given single-particle state, labelled by i, when the system is in equilibrium with a particle reservoir with temperature T and chemical potential μ.

2.b) Making use of the probability distribution(s) found in **2.a)**, find the average occupation number $\langle n_i \rangle$ and express P_i as a function of $\langle n_i \rangle$.