PHY 5524: Statistical Mechanics

March 2^{nd}, 2011
Assignment # 8
(Due Wednesday March 16^{th}, 2011)

Problem 1

The thermodynamic potential (often called the *Landau* or *grand potential*) for a Bose/Fermi system is given by,

$$Q(\mu, T, V)_{B/F} = \pm k_B T \sum_{\mathbf{k}} \ln \left[1 \mp e^{-(E_{\mathbf{k}}-\mu)/k_B T}\right] \ ,$$

where the sum is over all momentum states $\mathbf{k}$.

1.a) Compute the Bose/Fermi occupation number $n(\mathbf{k})$.

1.b) Obtain the classical limit of the thermodynamic potential by demanding that the occupancy of each momentum state $\mathbf{k}$ be $n(\mathbf{k}) \ll 1$. Show that in this limit the distinction between a Bose and a Fermi system disappears and explain what restriction does the classical limit pose on the fugacity $z \equiv \exp(\mu/k_B T)$.

1.c) Using the classical limit of the thermodynamic potential, obtain the average number of particles $\langle N \rangle$ in the system (in terms of μ, T, and V) by assuming a dispersion relation of the form

$$E_{\mathbf{k}} = A|\mathbf{k}|^n \equiv Ak^n \ , \quad (n = 1, 2, 3, \ldots) \ ,$$

where A is a positive constant.

1.d) Assuming that $\langle N \rangle = N =$ constant, compute the energy per particle of the system in the classical limit as a function of N, T, and V. If necessary, invert the equation obtained in (**1.c**) to compute the chemical potential μ (or the fugacity) in terms of N, T, and V.

1.e) Obtain the equation of state of the system (namely, the pressure in terms of N, T, and V) in the classical limit and show that it is independent of n.

1.f) Show that $E/PV = C_n =$ constant, and compute C_n.

Note: you might find the following expression useful,

$$\Gamma(z) \equiv \int_0^\infty x^{z-1} e^{-x} dx \quad \text{and} \quad \Gamma(z+1) = z\Gamma(z) \ .$$