PHY 5246: Theoretical Dynamics, Fall 2011

October 14^{th}, 2011
Assignment # 6
(Graded problems are due Friday October 21^{st}, 2011)

1 Graded problems

1. Discuss the motion of a particle in a central inverse-square law force field for a super-imposed force whose magnitude is inversely proportional to the cube of the distance from the particle to the center of force, that is

$$F(r) = -\frac{k}{r^2} - \frac{\lambda}{r^3} \quad k, \lambda > 0 \ .$$

Show that the motion is described by a precessing ellipse. Consider the cases $\lambda < l^2/m$, $\lambda = l^2/m$, and $\lambda > l^2/m$.

2. Chapter 3, Problem 13 of your Textbook.

3. A particle is moving in a potential

$$V(r) = -\frac{C}{3r^3} \quad (C > 0) \ .$$

(3.a) Given l (angular momentum), find the maximum value of the effective potential.

(3.b) Let the particle come in from infinity with speed v_0 and impact parameter b. In terms of C, m, and v_0, what is the largest value of b (call it b_{max}) for which the particle is captured by the potential? In other words, what is the *cross section* for capture, πb_{max}^2, for this potential?

4. A particle of mass m travels in a hyperbolic orbit past a mass M, whose position is assumed to be fixed. The speed at infinity is v_0, and the impact parameter is b.

(4.a) Show that the angle through which the particle is deflected is

$$\Theta = \pi - 2\tan^{-1}(\gamma b) \quad \Rightarrow \quad b = \frac{1}{\gamma}\cot\left(\frac{\Theta}{2}\right) \ ,$$

where $\gamma \equiv v_0^2/(GM)$.

(4.b) Let $d\sigma$ be the cross-sectional area (measured when the particle is initially at infinity) that gets deflected into a solid angle of size $d\Omega$ at angle Θ (this quantity is called *differential cross section*). Show that

$$\frac{d\sigma}{d\Omega} = \frac{1}{4\gamma^2 \sin^4(\Theta/2)} \ .$$

(4.c) Consider the case of *backward scattering*, i.e. $\Theta \approx 180°$. What can you tell in the limiting cases of small v_0 ($v_0 \to 0$) and large v_0 ($v_0 \to \infty$)? Explain your results.

(4.d) Consider the case of negligible deflection, i.e. $\Theta \approx 0°$. Does it make sense that $\sigma \approx \infty$ and why? How should the potential behave in order not to generate an infinite cross section?

(4.e) Show that in case you replace the gravitational force with the electrostatic one (Coulomb interaction between pointlike charges) you get Rutherford-scattering differential cross section:

$$\frac{d\sigma}{d\Omega} = \frac{K^2 q_1^2 q_2^2}{16E^2 \sin^4(\Theta/2)} \quad .$$

2 Non-graded suggested problems

5. Chapter 3, Problem 31 of your Textbook.

6. Chapter 3, Problem 32 of your Textbook.