PHY 5246: Theoretical Dynamics, Fall 2011

October 28^{th}, 2011
Extra practice problems

1. Consider a thin disk composed of two homogeneous halves connected along a diameter of the disk. If one half has density ρ and the other has density 2ρ, find the expression for the Lagrangian when the disk rolls without slipping along a horizontal surface (the rotation takes place in the plane of the disk).

2. A uniform solid cylinder of mass m, length b, and radius a is thrown up in the air; at the instant it is released ($t = 0$) it rotates with angular velocity $\omega = |\boldsymbol{\omega}|$about an axis that passes through its center and the outside edge of one end.

(2.a) What is the torque on the cylinder about its center of mass due to gravity?

(2.b) Write down Euler's equations for the cylinder, and so find the frequency of rotation about the central longitudinal axis, and the frequency at which the angular velocity $\boldsymbol{\omega}$ precesses about this axis in the body coordinate system.

(2.c) If the center of mass is initially moving straight upward at speed V_0, find the total kinetic energy at later times t.

3. A rigid body is made up of eight equal masses m at the corners of a wire frame with dimensions $2l(x) \times 2l(y) \times 4l(z)$. Take a body coordinate system with origin at the body center of mass. Imagine that the body is rotating with an angular velocity $\boldsymbol{\omega}$ that goes through a corner of the wire frame.

(3.a) If this angular velocity is constant, what happens to **L** in the body frame? How does **L** move in the fixed frame?

(3.b) Find the torque (expressed in the body system) required to maintain the given angular velocity $\boldsymbol{\omega}$.