PHY 5524: Statistical Mechanics

January 11^{th}, 2012

Assignment # 2

(Due Wednesday January 18^{th}, 2012)

Problem 1

Consider the problem of an isolated particle of mass m moving freely in a one-dimensional box of size L. Such a particle satisfies the following one-dimensional Schrödinger equation:

$$-\frac{\hbar^2}{2m}\frac{d^2\varphi(x)}{dx^2} = \epsilon\varphi(x) \ , \ \text{with} \ \varphi(0) = \varphi(L) = 0 \ .$$

1.a) Obtain the eigenvalues and the normalized eigenvectors of the Schrödinger equation.

Now consider an isolated system of 4 *non-interacting* particles of mass m placed in such one-dimensional box. The energy of the system equals $E = 63\epsilon_0$, where

$$\epsilon_0 = \frac{\hbar^2\pi^2}{2mL^2} \ ,$$

is the lowest eigenvalue of the Schrödinger equation. Find the entropy of the system for the following cases:

1.b) a system of 4 *distinguishable* spinless particles;

1.c) a system of 4 *indistinguishable* spinless bosons;

1.d) a system of 4 *indistinguishable* spin-1/2 fermions.

Problem 2

Consider a system of N localized particles moving under the influence of a quantum, one-dimensional, harmonic-oscillator potential of frequency ω. The energy of the system is given by

$$E = \frac{1}{2}N\hbar\omega + M\hbar\omega \ ,$$

where M is the total number of quanta in the system. That is,

$$M = \sum_{i=1}^{N} n_i \ ,$$

with $n_i = 0, 1, 2, \ldots$ representing the number of quanta in the i_{th} harmonic oscillator.

2.a) Compute the number of microstates Γ as a function of N and M.

2.b) Using Stirling's approximation, compute the entropy of the system as a function of N and M.

2.c) Compute the temperature T of the system as a function of N and M. Are there any values of N and M for which the temperature T becomes negative?

2.d) Compute the specific heat C_v of the system as a function of N and T. Note that in order to do so, you will need to express the total energy of the system as a function of N and T.

2.e) Compute the low- ($k_B T \ll \hbar\omega$) and high-temperature ($k_B T \gg \hbar\omega$) limits of the specific heat, and make a simple plot of its behavior as a function of temperature.

Problem 3

Problem 1.4 of Pathria's book.