PHY 5524: Statistical Mechanics

January 18^{th}, 2012
Assignment # 3
(Due Wednesday January 25^{th}, 2012)

Problem 1

Consider a system of N identical but distinguishable particles, each of which has two energy levels ($\pm\epsilon$). Using the microcanonical ensemble,

1.a) find the entropy of the system;

1.b) find the occupation numbers n_+ and n_- in terms of the temperature of the system;

1.c) explain how your result in point **(1.a)** (and **(1.b)**) would change if the upper energy level has a g-fold degeneracy, while the lower energy level is non-degenerate;

1.d) compute the Helmoltz free energy $F(T,N)$ as a function of temperature for case **(1.a)**;

1.e) explore the limits $T \to 0$ and $T \to \infty$ of the energy, the entropy, and the occupation numbers for case **(1.a)**;

1.f) can you explain why the maximum (minimum) entropy corresponds to the minimum (maximum) information on the system?

Problem 2

A simple picture of a rubber band is of a single long chain of links (you can think each link as a group of atoms) oriented in any direction. When the rubber band is pulled, so that he chain of links is completely linear, there is only one possible arrangement and the entropy is zero; when the rubber band is all tangled up there are a huge number of arrangements of the links (for a fixed length L) leading to a large entropy. If you heat up a rubber band under tension it will tangle up, while if you cool it down it will stretch.

You can build a simple statistical model of the rubber band by assuming that the links can lie only in two directions, either in the direction of increasing z or in the direction of decreasing z (assume for simplicity that z is your horizontal direction). Let there be n_+ links going to the right and n_- going to the left.

2.a) Find an expression for the entropy S of the rubber band as a function of its length L.

2.b) From the relation,

$$dU = T\,dS + F\,dl \ ,$$

you can get that,

$$\frac{F}{T} = -\left(\frac{\partial S}{\partial L}\right)_U \quad ,$$

where U is the internal energy, T is the temperature, F is the force of tension acting on the rubber band, and dl is an infinitesimal stretching of the rubber band itself. Use this relation to calculate the expression of the force of tension.

2.c) Can you explain the sort of counter-intuitive result that under constant tension the rubber band contracts when you heat it up, while it relaxes when you cool it down?

Problem 3

Problem 1.7 of Pathria's book.

Problem 4

Problem 2.1 of Pathria's book.