PHY 5524: Statistical Mechanics

February 13th, 2012

Assignment # 6

(Due Wednesday February 15th, 2012)

Problem 1

We have shown in class that in the canonical ensemble, where the energy of the system fluctuates, the mean-square fluctuations in the energy is related to the specific heat at constant volume. In the grand-canonical ensemble both the energy and the number of particles are allowed to fluctuate.

1.a) Show that in the grand-canonical ensemble the mean-square fluctuations in the number of particles may be written as,

$$(\Delta N)^2 \equiv \langle N^2 \rangle - \langle N \rangle^2 = k_B T \left(\frac{\partial \langle N \rangle}{\partial \mu} \right)_{T,V} \quad .$$

1.b) Using suitable thermodynamics identities, show that the *fractional* mean-square fluctuations in the number of particles is given by the following expression,

$$\frac{(\Delta N)^2}{\langle N \rangle^2} = \frac{k_B T}{V} K_T \quad ,$$

where

$$K_T = -\frac{1}{V} \left(\frac{\partial V}{\partial P} \right)_T \quad ,$$

is called the *isothermal compressibility* of the system.

1.c) Evaluate the r.h.s. of the above expression for a classical ideal gas and conclude that the fractional root-mean-square fluctuations in the number of particles are proportional to $1/\sqrt{\langle N \rangle}$. That is,

$$\frac{\sqrt{(\Delta N)^2}}{\langle N \rangle} \propto \frac{1}{\sqrt{\langle N \rangle}} \quad .$$