PHY 5524: Statistical Mechanics

February 15^{th}, 2012
Assignment # 7
(Due Wednesday February 22^{nd}, 2012)

Start reading sections 5.4, 6.1-6.2 of Pathria's book (you will need only parts of them) and think of how to approach the following problems. We will discuss both problems on Monday, February 20th, either during lesson or during the following problem session.

Problem 1

Two non-interacting particles inhabit a potential well such that the orbital motion of each particle gives rise to an energy spectrum $E(n) = n\epsilon$, the n^{th} energy level having a degeneracy $g(n) = 2n + 1$. Consider spin-0 particles obeying Bose-Einstein statistics, spin-$\frac{1}{2}$ particles obeying Fermi-Dirac statistics, and spin-s particles obeying Maxwell-Boltzmann statistics. Assuming that the energy is spin-independent,

1.a) find the microcanonical partition function of the system when the total energy has the fixed value $E = N\epsilon$,

1.b) find the canonical partition function of the system when it is in contact with a heat reservoir at fixed temperature T, and

1.c) investigate the relationship between the partition functions obtained for the three types of statistics.

Problem 2

Consider a system of non-interacting indistinguishable particles whose states can be specified in the following way.

(i) There are single particle states, labelled by an index i, with energy ϵ_i, which will be degenerate (ϵ_i may have the same value for several values of i) for particles with non-zero spin.

(ii) Each multiparticle state corresponds to a set of occupation numbers $\{n_i\}$, where n_i counts the number of particles occupying the i^{th} single-particle state and has values from 0 to M. Each distinct set of occupation numbers corresponds to a distinct state.

Use the grand-canonical ensemble, and consider separately the cases in which the system obeys Bose-Einstein, Fermi-Dirac, or Maxwell-Boltzmann statistics.

2.a) Obtain the probability distribution(s) $P_i(n_i, T, \mu)$ for finding n_i particles in a given single-particle state, labelled by i, when the system is in equilibrium with a particle reservoir with temperature T and chemical potential μ.

2.b) Making use of the probability distribution(s) found in **2.a)**, find the average occupation number $\langle n_i \rangle$ and express P_i as a function of $\langle n_i \rangle$.