1) Origins of SUSY breaking

A) General considerations for spontaneous SUSY breaking

By definition, spontaneously broken SUSY means that the vacuum state $|0\rangle$ is not invariant under supersymmetry transformations, so $Q_\alpha |0\rangle \neq 0$ and $Q_{\dot\alpha}^\dagger |0\rangle \neq 0$.

→ In global susy, the Hamiltonian H related to the SUSY generators Q is given by —

$$H = P^0 = \frac{1}{4}\left(Q_1 Q_1^\dagger + Q_1^\dagger Q_1 + Q_2 Q_2^\dagger + Q_2^\dagger Q_2\right) \qquad \text{— ①}$$

If SUSY is unbroken, we should get $H|0\rangle = 0$ and vacuum has zero energy.

Conversely, if SUSY is spontaneously broken in the vacuum state, the vacuum must have positive energy, since

$$\langle 0|H|0\rangle = \frac{1}{4}\left(\| Q_1^\dagger |0\rangle\|^2 + \| Q_1 |0\rangle\|^2 + \| Q_2^\dagger |0\rangle\|^2 + \| Q_2 |0\rangle\|^2\right) > 0.$$

if the Hilbert space is to have positive norm.

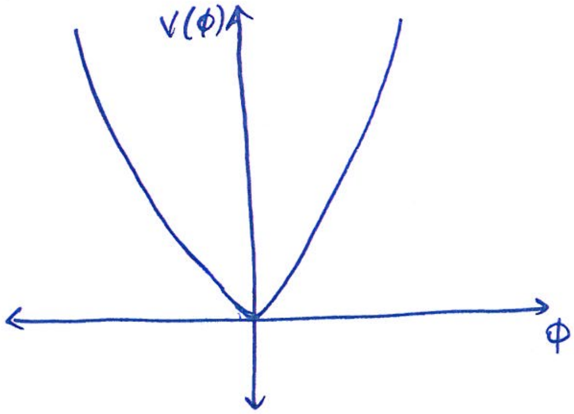
If space-time dependent effects and fermion condensates can be neglected, then $\langle 0|H|0\rangle = \langle 0|V|0\rangle$, where V is the scalar potential

$$V(\phi, \phi^*) = F^{*i} F_i + \frac{1}{2}\sum_a D^a D^a = W_i^* W^i + \frac{1}{2}\sum_a g_a^2 (\phi^* T^a \phi)^2 \qquad \text{— ②}$$

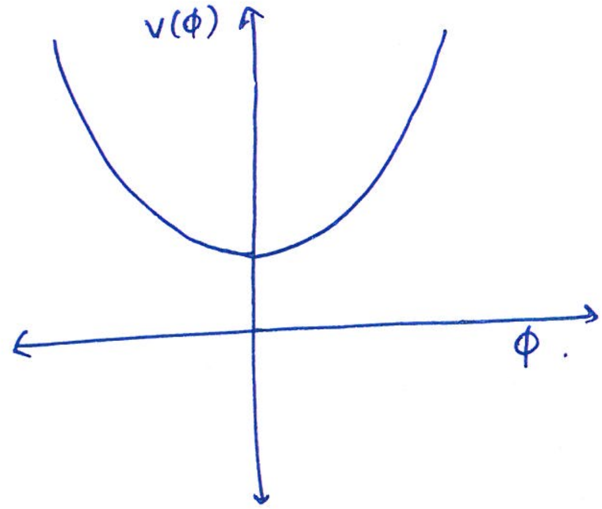
here eq.ⁿ of motion for D^a is coming from $\mathcal{L}_{gauge}$

$$D^a = -g(\phi^* T^a \phi).$$

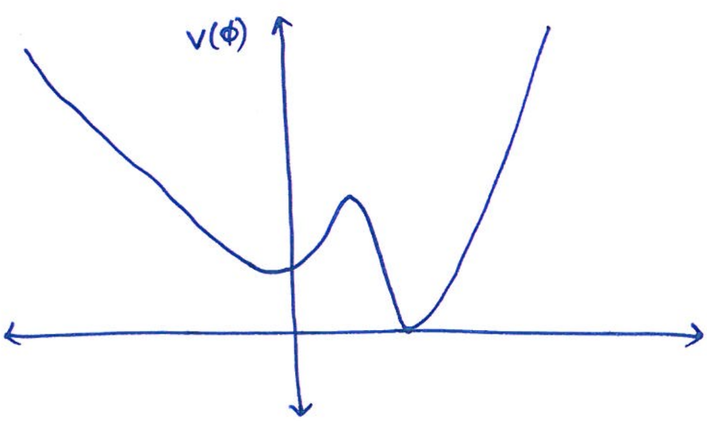
It is clear that SUSY will be spontaneously broken if the expectation value of F_i and/or D^a does not vanish in the vacuum state. One way to guarantee spontaneous symmetry breaking is to look for models in which $F_i = 0$ and $D^a = 0$ cannot all be simultaneously satisfied for any values of the fields.



Scalar potential for unbroken supersymmetry



spontaneously broken SUSY.



← here we don't live in a true ground state, but we live in a metastable ground state, lifetime at least of order of the present age of the universe.

metastable supersymmetry breaking.

B) Fayet - Iliopoulos (D-term) SUSY breaking :

This uses a non-zero D-term for a $U(1)$ gauge group. The idea is to add a term linear in the auxiliary field to the theory :

$$\mathcal{L}_{FI} = -kD \quad — \quad (3)$$

where k is a constant with dimensions of $[\text{mass}]^2$.

This term is gauge-invariant and supersymmetric by itself. Now the relevant part of the scalar potential

$$V = kD - \frac{1}{2}D^2 - gD\sum_i q_i |\phi_i|^2 .$$

[coming from $\mathcal{L}_{gauge} = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu a} + i\lambda^{\dagger a}\bar{\sigma}^\mu \nabla_\mu \lambda^a + \frac{1}{2}D^a D^a$.

and $\mathcal{L} = \mathcal{L}_{chiral} + \mathcal{L}_{gauge}$
$- \sqrt{2}g(\phi^* T^a \psi)\lambda^a - \sqrt{2}g\lambda^{\dagger a}(\psi^\dagger T^a \phi) + g(\phi^* T^a \phi)D^a$

Supersymmetric gauge interaction]

here q_i are the charges of the scalar fields ϕ_i under the $U(1)$ gauge group in question.

So, equation of motion :

$$D = k - g\sum_i q_i |\phi_i|^2 \quad — \quad (4)$$

Now, suppose that the scalar fields ϕ_i that are charged under the U(1) all have non-zero superpotential masses m_i. (Gauge invariance then requires that they come in pairs with opposite charges.)

So, potential will have the form:

$$V = \sum_i |m_i|^2 |\phi_i|^2 + \frac{1}{2}\left(k - g \sum_i q_i |\phi_i|^2\right)^2$$

Since this cannot vanish, supersymmetry must be broken; one can check that the minimum always occurs for non-zero D.

For the simplest case in which $|m_i|^2 > g q_i k$ for each i, the minimum is realized for all $\phi_i = 0$ and $D = k$. with the U(1) gauge symmetry unbroken.

For non-Abelian gauge groups, the analog of eqn. 3 would not be gauge-invariant and is therefore not allowed, so only U(1) D-terms can drive spontaneous symmetry breaking. Unfortunately this cannot work, because the squarks and sleptons do not have superpotential mass terms. So, at least some of them would just get non-zero VEVs in order to make eqn 4 vanish. That would break color and/or electromagnetism, but not supersymmetry. Therefore, a Fayet-Iliopoulos term for $U(1)_Y$ must be subdominant compared to other sources of SUSY breaking.

If this is a the dominant source of SUSY breaking it proves difficult to give appropriate masses to all the MSSM particles, especially the gauginos.

C ~~B~~) O'Raifeartaigh (F-term) supersymmetry breaking :

Models where spontaneous ~~n~~ SUSY breaking is ultimately due to a non-zero F-term VEV, called O'Raifeartaigh models, have brighter phenomenological prospects.

Here the idea is to pick a set of chiral super multiplets $\Phi_i \supset (\phi_i, \psi_i, F_i)$ and a superpotential W in such a way that the eq^{n}s $F_i = -\frac{\delta W^*}{\delta \phi^{*i}} = 0$ have no simultaneous solution within some compact domain. Then $V = \sum_i |F_i|^2$ will have to ~~be~~ be positive at its minimum, ensuring that SUSY is broken.

→ The simplest example with a supersymmetry breaking global minimum has three chiral supermultiplets $\Phi_{1,2,3}$ with superpotential :

$$W = -k\Phi_1 + m\Phi_2\Phi_3 + \frac{y}{2}\Phi_1\Phi_3^2$$

W contains a linear term, with k having dimensions of [mass]^2. Such a term is allowed if the corresponding chiral supermultiplet is a gauge singlet. A linear term is necessary to achieve F-term breaking at tree-level in renormalizable superpotentials, since otherwise setting all $\phi_i = 0$ will always give a supersymmetric global minimum with all $F_i = 0$.

$$F_1 = -\left(\frac{\partial W}{\partial \phi_1}\right)^* = -\left(-k + \frac{y}{2}\phi_3^{*2}\right)$$

$$F_2 = -\left(\frac{\partial W}{\partial \phi_2}\right)^* = -m\,\phi_3^*$$

$$F_3 = -\left(\frac{\partial W}{\partial \phi_3}\right)^* = -\left(m\,\phi_2^* + y\,\phi_1^*\phi_3^*\right)$$

here $V_{\text{tree-level}} = |F_1|^2 + |F_2|^2 + |F_3|^2$

here we see, $F_1 = 0$ and $F_2 = 0$ are not compatible, so SUSY must indeed be broken. If $m^2 > yk$, then the absolute minimum of the classical potential is at $\phi_2 = \phi_3 = 0$ with ϕ_1 undetermined, so $F_1 = k$ and $V_{\text{tree-level}} = k^2$ at the minimum.

The fact that ϕ_1 is undetermined at the tree-level is an example of a "flat direction" in the scalar potential

The flat direction parameterized by ϕ_1 is an accidental feature of the classical scalar potential and in this case, it will be lifted by quantum corrections.

D) Need for SUSY breaking Hidden sector:

Fayet-Iliopoulos and O'Raifeartaigh models set the scale of SUSY breaking by a dimensionful parameter which is put in by hand. To get a SUSY breaking scale naturally small compared to the Planck scale, M_{Pl}, we essentially need an asymptotically free gauge theory that gets strong through RG evolution at some much smaller scale:

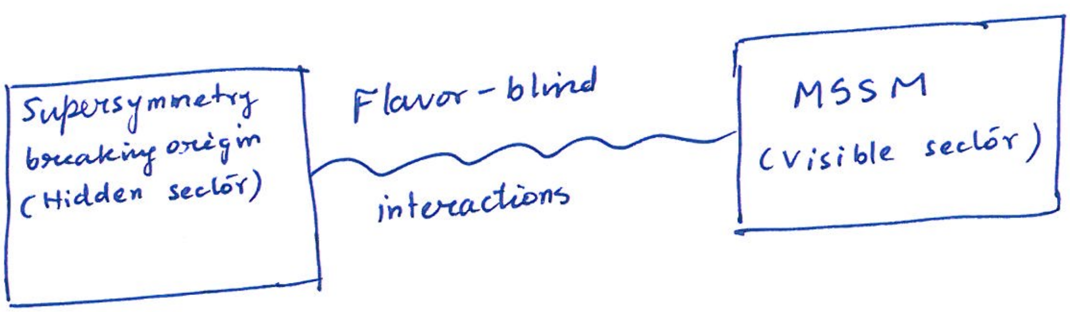
$$\Lambda \sim e^{-8\pi^2/(b g_0^2)} M_{Pl}$$

g_0 is the running gauge coupling at M_{Pl} with negative beta function $-|b|\, g^3/16\pi^2$

This breaks SUSY non perturbatively.

Again, we cannot rely on renormalizable tree-level couplings to directly transmit SUSY breaking to the MSSM fields, since, first, SUSY does not allow scalar-gaugino-gaugino couplings. Second, at least some of the MSSM squarks and sleptons would have to be unacceptably light mass and should have been discovered already.

Hence, we expect that SUSY breaking occurs dynamically in a "hidden" sector and is communicated by non-renormalizable interactions or through loop effects.



Supersymmetry breaking evidently occurs in a "hidden sector" of particles that have no or only very small direct couplings to the "visible sector" chiral supermultiplets of the MSSM. However the two sectors do share some interactions that are responsible for mediating soft SUSY breaking from the hidden sector to the visible sector, resulting in the MSSM soft terms.

[In this scenario, the tree-level squared mass sum rules need not hold, even approximately, for the physical masses of the visible sector fields, so that a phenomenologically viable superpartner mass spectrum is, in principle, achievable.]

There have been two main competing proposals for what the mediating interactions might be.

1) The first is that they are gravitational. They are associated with the new physics including gravity, that enters near the Planck scale. In this "gravity"-mediated" or "Planck Scale mediated SUSY breaking (PMSB)" scenario, if SUSY is broken in the hidden sector by a VEV $\langle F \rangle$, then the soft terms in the visible sector should be roughly:

$$m_{soft} \sim \frac{\langle F \rangle}{M_P} \qquad \text{⑤}$$

(dimensional analysis)

This is because we know that m_{soft} must vanish in the limit $\langle F \rangle \to 0$ where SUSY is unbroken, and also in the limit $M_P \to \infty$ (corresponding to $G_{Newton} \to 0$) in which gravity becomes irrelevant.

So, for $m_{soft} \sim$ a few hundred GeV,

$$\sqrt{\langle F \rangle} \sim 10^{10} \text{ or } 10^{11} \text{ GeV}.$$

↑

the scale associated with the origin of SUSY breaking in the hidden sector.

2) A second possibility is that the flavor-blind mediating interactions for supersymmetry breaking ~~GMSB~~ are the ordinary electroweak and QCD gauge interactions. In this gauge-mediated supersymmetry breaking (GMSB) scenario, the MSSM soft terms come from loop diagrams involving some messenger particles. The messengers are new chiral supermultiplets that couple to a ~~susp~~ SUSY-breaking VEV $\langle F \rangle$, and also have $SU(3)_c \times SU(2)_L \times U(1)_Y$ interactions, which provide necessary connection to the MSSM.

Then MSSM soft terms:

$$m_{soft} \sim \frac{\alpha_a}{4\pi} \frac{\langle F \rangle}{M_{mess}}.$$

$\frac{\alpha_a}{4\pi}$ is a loop factor for Feynman diagrams involving gauge interactions, and M_{mess} is a characteristic scale of the masses of the messenger fields.

If M_{mess} and $\sqrt{\langle F \rangle}$ are roughly comparable, the scale of SUSY breaking can be as low as about $\sqrt{\langle F \rangle} \sim 10^4$ GeV (much-lower than in the gravity-mediated case!) to give m_{soft} of the right order of magnitude.

E) The goldstino and the gravitino:

The spontaneous breaking of global supersymmetry implies the existence of a massless Weyl fermion, the goldstino. The goldstino is the fermionic component of the supermultiplet whose auxiliary field obtains a VEV.

Suppose that the only non-vanishing auxiliary field VEV is $\langle F \rangle$ with goldstino superpartner $\tilde{G}$. Then the supercurrent conservation equation tells us that —

$$0 = \partial_\mu J^\mu_\alpha = -i \langle F \rangle (\sigma^\mu \partial_\mu \tilde{G}^\dagger)_\alpha + \partial_\mu j_\alpha^\mu + \dots$$

where j_α^μ is the part of the supercurrent that involves all of the other supermultiplets, and the ellipses represent other contributions of the goldstino supermultiplet to $\partial_\mu J_\alpha^\mu$, which we can ignore.

Effective Lagrangian:

$$\mathcal{L}_{\text{goldstino}} = i \tilde{G}^\dagger \bar{\sigma}^\mu \partial_\mu \tilde{G} - \frac{1}{\langle F \rangle} (\tilde{G} \partial_\mu j^\mu + c.c.) \quad \text{— (6)}$$

which describes the interactions of the goldstino with all other fermion-boson pairs.

$$j_\alpha^\mu = (\sigma^\nu \bar{\sigma}^\mu \psi_i)_\alpha \partial_\nu \phi^{*i} - \sigma^\nu \bar{\sigma}^\rho \sigma^\mu \lambda^{\dagger a} F^a_{\nu\rho} / 2\sqrt{2} + \dots$$

there are goldstino - scalar - chiral fermion and goldstino - gaugino - gauge boson vertices:

Superpartner pairs (ϕ, ψ) and (λ, A)

This derivation depends only on supercurrent conservation
⑥ holds independently of the details of how SUSY breaking is communicated from $\langle F \rangle$ to the MSSM sector fields (ϕ_i, ψ_i) and (λ^a, A^a).

When $\langle F \rangle \to 0$, the interaction couplings get larger! But wait. Since $\tilde{a}\, \partial_\mu j^\mu$ contain two derivatives this will always give a kinematic factor proportional to the squared-mass difference of the superpartners when they are on-shell, so $m_\phi^2 - m_\psi^2$ and $m_\lambda^2 - m_A^2$.
These can be non-zero only by virtue of SUSY breaking, hence as $\langle F \rangle \to 0$, they must also vanish.

<u>Supergravity</u>: Taking into account gravity, supersymmetry must be promoted to a local symmetry.
So, spinor ~~aa~~ parameter ϵ^α is no longer constant.
It unifies the spacetime symmetries of ordinary general relativity with local supersymmetry transformations.
In supergravity, spin-2 graviton has a spin-3/2 fermion superpartner called the gravitino ~~$(\tilde{\Psi}^\alpha_\alpha)$~~ $(\tilde{\Psi}^\alpha_\mu)$

$\mu \to$ vector index
$\alpha \to$ spinor index.

$$\delta \tilde{\Psi}^\alpha_\mu = \partial_\mu \epsilon^\alpha + \ldots$$

gravitino should be thought of as the "gauge" field of local supersymmetry transformations.

As long as supersymmetry is unbroken, the graviton and the gravitino are both massless (each with 2-helicity states).

Once SUSY is spontaneously broken, the gravitino acquires a mass by absorbing the goldstino which becomes its longitudinal (helicity $\pm 1/2$) components.

This is super-Higgs mechanism.

Massive spin-3/2 gravitino now has 4 helicity states, of which two ~~are~~ were originally assigned to the would-be goldstino.

gravitino mass $m_{3/2} \sim \dfrac{\langle F \rangle}{M_P}$

when $\langle F \rangle \to 0$ $\quad m_{3/2} \to 0$
(SUSY unbroken)

when $M_P \to \infty$ $\quad m_{3/2} \to 0$
(Gravity turned off)

$\to$ In Planck-scale-mediated SUSY breaking case, the gravitino mass is comparable to the masses of the MSSM sparticles. Here $m_{3/2} \sim 100$ GeV.

Its interaction will be gravitational : so it will not play any role in collider physics.

→ Gauge - mediated SUSY breaking models predict that gravitino is much lighter than MSSM sparticles as long as $M_{mess} \ll M_P$.

[this is because $m_{soft} \sim \frac{\alpha_a}{4\pi} \frac{\langle F \rangle}{M_{mess}}$

and $m_{3/2} \sim \frac{\langle F \rangle}{M_P}$]

The gravitino is almost certainly the LSP, and all of the MSSM sparticles will eventually decay into final states of that include it.

Gravitino has inherits the non-gravitational interactions of the goldstino it has absorbed. So gravitino's longitudinal component can play an important role in collider physics experiment.

The mass and transverse (helicity $\pm 3/2$) components can generally be ignored for kinematic ~~processes~~ purposes, ~~specially~~ so in collider phenomenology we can interchangeably use $\tilde{G}$ for the gravitino and goldstino.

⑤ D
Susy-breaking.

The decay rate of any sparticle $\tilde{X}$ into its SM partner X plus a $\tilde{G}$ is —

$$\Gamma(\tilde{X} \to X\tilde{G}) = \frac{m_{\tilde{X}}^5}{16\pi \langle F\rangle^2}\left(1 - \frac{m_X^2}{m_{\tilde{X}}^2}\right)^4 .$$

$$(\tilde{X}, X) = (\phi, \psi) \text{ or } (\lambda, A).$$

one $\left(1 - \frac{m_X^2}{m_{\tilde{X}}^2}\right)^2$ comes from derivatives in the interaction term and another one comes from kinematic phase space integral.

[If supermultiplet ~~containing~~ containing goldstino has canonically normalized kinetic terms and $\langle F\rangle$ has and the tree level vacuum energy is required to vanish, then $m_{3/2} = \frac{\langle F\rangle}{\sqrt{3}M_P}$.

Then $\Gamma(\tilde{X} \to X\tilde{G}) = \frac{m_{\tilde{X}}^5}{48\pi M_P^2 m_{3/2}^2}\left(1 - \frac{m_X^2}{m_{\tilde{X}}^2}\right)^4$

If $m_{\tilde{X}} \sim 100$ GeV and $\langle F\rangle \lesssim 10^6$ GeV.,

then ~~I~~ the decay can occur quickly enough to be observed in a modern collider.]

F) ~~Ea~~ Gauge-mediated SUSY-breaking models :

The basic idea is to introduce some new chiral supermultiplets called messengers, that couple to the ultimate source of SUSY breaking, and also couple indirectly to the (s)quarks and (s)leptons and Higgs(ino) of the MSSM through the ordinary $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge boson and gaugino interactions.

There is still gravitational communication betn. the MSSM and the source of SUSY breaking, but that effect is relatively unimportant compared to the gauge interaction effects.

In the simplest model, messenger fields are left-handed chiral supermultiplets $q, \bar{q}, l, \bar{l}$ transforming under $SU(3)_c \times SU(2)_L \times U(1)_Y$ as:

$q \sim (3, 1, -\frac{1}{3})$, $\bar{q} \sim (\bar{3}, 1, \frac{1}{3})$, $l \sim (1, 2, \frac{1}{2})$, $\bar{l} \sim (1, 2, -\frac{1}{2})$.

Supermultiplets contain messenger quarks $\psi_q, \psi_{\bar{q}}$, scalar quarks $q, \bar{q}$, messenger leptons $\psi_l, \psi_{\bar{l}}$ and scalar leptons $l, \bar{l}$.

Assume they get heavy masses by coupling to a gauge-singlet chiral supermultiplets S through a superpotential:

$$W_{mess} = y_2 S \ell \bar{\ell} + y_3 S q \bar{q}$$

Scalar component of S and its auxiliary component F are supposed to acquire VEVs, $\langle S \rangle$ and $\langle F \rangle$.

We will simply parameterize our ignorance of the precise mechanism of SUSY breaking by asserting that S participates in another part of the superpotential:

$W_{breaking}$

↑

Provides necessary spontaneous breaking of SUSY.

Fermionic messenger fields pair up to get mass terms:

$$\mathcal{L} = -y_2 \langle S \rangle \psi_\ell \psi_{\bar{\ell}} - y_3 \langle S \rangle \psi_q \psi_{\bar{q}} + c.c.$$

Then $$V = \left| \frac{\delta W_{mess}}{\delta \ell} \right|^2 + \left| \frac{\delta W_{mess}}{\delta \bar{\ell}} \right|^2 + \left| \frac{\delta W_{mess}}{\delta q} \right|^2 + \left| \frac{\delta W_{mess}}{\delta \bar{q}} \right|^2$$

$$+ \left| \frac{\delta}{\delta S} \left(W_{mess} + W_{brking} \right) \right|^2$$

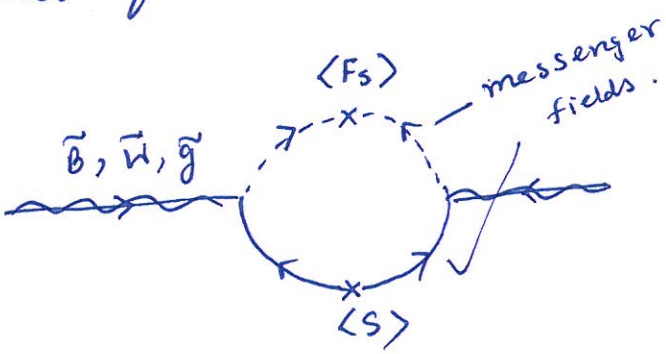
Effect of SUSY breaking is to split each messenger super-multiplet pair apart:

$l, \bar{l}$: $m^2_{fermions} = |y_2 \langle S \rangle|^2$, $m^2_{scalars} = |y_2 \langle S \rangle|^2 \pm |y_2 \langle F_S \rangle|$

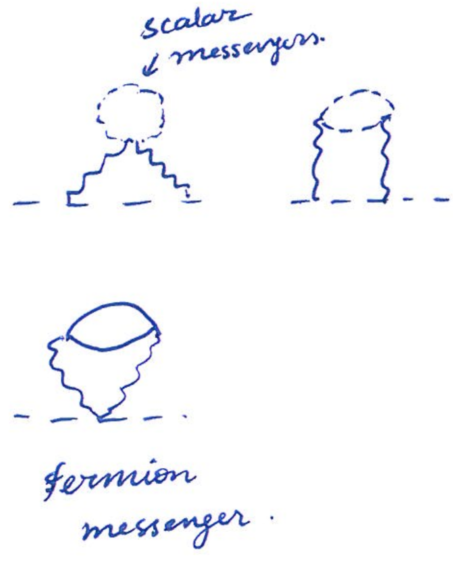
$q, \bar{q}$: $m^2_{fermions} = |y_3 \langle S \rangle|^2$, $m^2_{scalars} = |y_3 \langle S \rangle|^2 \pm |y_3 \langle F_S \rangle|$

→ The supersymmetry violation apparent in this messenger spectrum for $\langle F_S \rangle \neq 0$ is communicated to the MSSM sparticles through radiative corrections.

→ The scalars of the MSSM do not get any radiative corrections to their masses at one-loop order. The leading contribution to their masses comes from the 2-loop graphs



Contribution to the MSSM gaugino masses



Suppose at minimum of the potential:

$$\langle S \rangle \neq 0$$

$$\left\langle \frac{\delta W_{breaking}}{\delta S} \right\rangle = -\langle F_S^* \rangle \neq 0$$

$$\left\langle \frac{\delta W_{mess}}{\delta S} \right\rangle = 0 .$$

quadratic mass terms in the potential for the messenger scalar leptons:

$$V = |y_2 \langle S \rangle|^2 (|\ell|^2 + |\bar{\ell}|^2) + |y_3 \langle S \rangle|^2 (|q|^2 + |\bar{q}|^2)$$

$$-(y_2 \langle F_S \rangle \ell \bar{\ell} + y_3 \langle F_S \rangle q \bar{q} + c.c.)$$

+ quartic terms.

The complex scalar messengers $\ell, \bar{\ell}$ obtain a squared mass matrix:

$$\begin{pmatrix} |y_2 \langle S \rangle|^2 & -y_2^* \langle F_S^* \rangle \\ -y_2 \langle F_S \rangle & |y_2 \langle S \rangle|^2 \end{pmatrix}$$

squared mass eigenvalues: $|y_2 \langle S \rangle|^2 \pm |y_2 \langle F_S \rangle|$.

scalars get squared masses $|y_3 \langle S \rangle|^2 \pm |y_3 \langle F_S \rangle|$.

G) Planck-scale-mediated supersymmetry breaking models

In early 1980s, when gravitation was first considered as the mediator of SUSY breaking, it was generally assumed that superpotential consisted of two terms:

$f(\Phi) \rightarrow$ various left-chiral superfields Φ_r of an observable sector. including all the super fields of observable particles.

$\tilde{f}(Z) \rightarrow$ various left-chiral superfields Z_k of a hidden sector.

All of which are neutral under the $SU(3) \times SU(2) \times U(1)$ gauge group of the SM. Further the superpotential of the hidden sector was assumed to take to form:

$$\tilde{f}(Z) = \epsilon^3 F(\kappa Z).$$

$\epsilon << M_{PL}$,

$F(\kappa Z)$ is power series in κZ with coeff. of the order of unity.

Total superpotential : $f(\Phi) + \tilde{f}(Z)$.

$\longrightarrow$ serious criticism of this approach is that it did not offer any hope of solving the hierarchy problem