PHY 5667 : Quantum Field Theory A, Fall 2014

November 26^{th}, 2014

Final Exam

(due Friday December 12^{th}, 2014)

This final exam is about *Scalar Electrodynamics*, which is introduced in Chapter 61 and calculated at the one-loop level in Chapter 65 of your textbook.

Scalar electrodynamics is the theory of a complex scalar field (spin-zero) with a quartic self-interaction, and a vector field (spin-one). The vector-field part is what will be new to you, although you already know almost everything from your background. Indeed, the Lagrangian of scalar electrodynamics is,

$$\mathcal{L} = -(D^\mu\varphi)^\dagger D_\mu\varphi - m^2\varphi^\dagger\varphi - \frac{1}{4}\lambda(\varphi^\dagger\varphi)^2 - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} \ , \tag{1}$$

where D_μ is the so-called *covariant derivative* defined as,

$$D_\mu \equiv \partial_\mu - ieA_\mu \ , \tag{2}$$

with e the charge of the electron, and A_μ the vector field in terms of which the $F_{\mu\nu}$ *field strength* tensor is defined as,

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \ , \tag{3}$$

as should be familiar to you from relativistic electrodynamics and from Homework 1 of this class. Looking at the Lagrangian in Eq. (1), you should recognize: the Lagrangian of a complex scalar field with quartic self-interaction, the Maxwell Lagrangian of a vector field A^μ, and their interaction (hidden in the $(D^\mu\varphi)^\dagger D_\mu\varphi$ term). This theory describes the interaction of spin-zero charged particles (hence the need for a complex scalar field) with *photons* (the excitations or *particles* corresponding to the vector field A^μ).

If you are curious about *Spinor Electrodynamics*, also known as *Quantum Electrodynamics* or *QED*, you should read Chapters 58-64 of your textbook. We will cover them in QFT B, and you do not need them for this exam, but they may help setting the context for you and fill in some background.

To complete this exam proceed as follows:

1. read Chapter 61 of your textbook;
2. make sure you understand Eq. (1) above;
3. make sure you derive and understand the Feynman rules given in Chapter 61 (you do not need to show any work for this part);
4. do problem 61.1 of your textbook;
5. do problem 61.2 of your textbook;
6. read Chapter 65 of your textbook;

7. do problem 65.1 of your textbook;

8. do problem 65.2 of your textbook;

9. do problem 66.2 of your textbook.

We will discuss this topic in class starting on Tuesday December, 2^{nd} till Friday December, 5^{th}. You are welcome to bring your questions to class and keep them ready for when we discuss the specific of the matter.