PHY 5669 : Quantum Field Theory B, Spring 2015

February 26^{th}, 2015
Assignment # 4
(due Thursday March 19^{th}, 2015)

1. Problem 62.1 of Srednicki's book.

2. Problem 62.2 of Srednicki's book.

3. Consider the one-loop diagrams with three external photons (and no external fermions) and show that the corresponding amplitude is zero. This is a particular case of *Furry's theorem*, for which you may want to solve Problem 58.2).

4. Problem 62.3 of Srednicki's book.

5. Problem 66.3 of Srednicki's book.

6. **Cancelation of IR divergences in inclusive cross sections**.
Consider the scattering process $e^-(p)+\gamma(q)\to e^-(p')$ ($p^2=(p')^2=-m^2$, $q^2\neq 0$) discussed in class, and show that the IR divergences arising in the $O(\alpha)$ QED loop corrections cancel in the $O(\alpha^2)$ inclusive cross section via equal and opposite terms present in the one-photon emission contributions. To this extent it can be useful to consider the following series of steps:

6.1 write the $O(\alpha^2)$ cross section as a sum of a tree-level piece (σ_0) and an $O(\alpha)$ correction (σ_1) (notice that σ_0 starts at $O(\alpha)$, and σ_1 is per se of $O(\alpha^2)$ but represents the $O(\alpha)$ correction to σ_0);

6.2 show that σ_1 receives contributions both from one-loop (or *virtual*) corrections to the $e^-(p)+\gamma(q)\to e^-(p')$ process (show/identify them explicitly) and from the tree-level process $e^-(p)+\gamma(q)\to e^-(p')+\gamma(k)$ (*real* photon emission, show diagrams explicitly), where the final state photon might not be identified, hence the meaning of *inclusive* cross section;

6.3 review and reproduce the calculation of the IR divergences present in the $O(\alpha^2)$ one-loop part of the cross section (I would like you to use dimensional regularization and to show how to calculate the IR-divergent parts, limiting yourself to the pole/singular parts only);

6.4 calculate the cross section for $e^-(p)+\gamma(q)\to e^-(p')+\gamma(k)$ in the limit of vanishing photon energy ($k^0\to 0$, a.k.a. *soft* limit) keeping only the IR divergent pieces. It is useful to notice that in this limit both the scattering amplitude and the phase-space integration factorize, such that,

$$\int dPS_{2\to 2}|A(e^-\gamma\to e^-\gamma)|^2 \stackrel{k^0\to 0}{\longrightarrow} \int dPS_{2\to 1}\int dPS_\gamma\, S_{\rm soft}\,|A(e^-\gamma\to e^-)|^2\ , \qquad (1)$$

where S_{soft} is given by,

$$S_{\text{soft}} = \frac{p^2}{(pk)^2} + \frac{(p')^2}{(p'k)^2} - \frac{2(p \cdot p')}{(p \cdot k)(p' \cdot k)} \ , \tag{2}$$

and the photon phase space in $d = 4 - 2\epsilon$ dimensions can be written as,

$$\int dPS_\gamma = \int \frac{d^{d-1}k}{(2\pi)^{d-1} 2k^0} = \frac{(4\pi)^\epsilon}{8\pi^2} \int_0^\delta dk^0 (k^0)^{1-2\epsilon} \int_0^\pi d\theta \sin^{1-2\epsilon}\theta \ , \tag{3}$$

where δ is an arbitrary upper bound on the photon energy.

6.5 You should now have all the information to answer the problem.