PHY 5246: Theoretical Dynamics, Fall 2015

August 26^{th}, 2015
Assignment # 1
(Due Wednesday September 2^{nd}, 2015)

1. Consider a planar motion. In Cartesian coordinates a general vector takes the form,

$$\mathbf{r} = x\hat{\mathbf{x}} + y\hat{\mathbf{y}} = r\cos\theta\hat{\mathbf{x}} + r\sin\theta\hat{\mathbf{y}} \ .$$

Rewrite the same vector in terms of polar basis vectors $\hat{\mathbf{r}}$ and $\hat{\theta}$, and calculate $\dot{\mathbf{r}}$ and $\ddot{\mathbf{r}}$ in the same basis.

2. Consider a charged particle entering a region of uniform magnetic field $\mathbf{B}$ (for example: the Earth's field). Choose a Cartesian coordinate system with one of the axis in the direction of the magnetic field. If q is the charge of the particle and $\mathbf{v}$ its velocity, the particle is subject to a force: $\mathbf{F} = q\mathbf{v} \times \mathbf{B}$. Determine its subsequent motion, i.e. write down the equations of motion (using 2nd Newton's law) and solve them.

3. Show that for a single particle with constant mass the equation of motion implies the following differential equation for the kinetic energy:

$$\frac{dT}{dt} = \mathbf{F} \cdot \mathbf{v} \ ,$$

while if the mass varies with time the corresponding equation is

$$\frac{d(mT)}{dt} = \mathbf{F} \cdot \mathbf{p} \ .$$

4. Suppose a system of two particles is known to obey the equations of motion in Eqs. (1.22) and (1.26), of your textbook, i.e.

$$M\ddot{\mathbf{R}} = \mathbf{F}^{(\mathbf{e})} \ ,$$

and

$$\dot{\mathbf{L}} = \mathbf{N}^{(\mathbf{e})} \ .$$

From the equations of motion of the individual particles show that the internal forces between particles satisfy both the weak and strong laws of action and reaction. The argument can be generalized to a system with an arbitrary number of particles, thus proving the converse of the arguments leading to the previous equations.

5. A light string of length a has bobs of mass m_1 and m_2 ($m_2 > m_1$) on its ends. The end of m_1 is held and whirled by hand above the head and then released. Describe the subsequent motion, and find the tension in the string after release (assume that the string stays straight during the whole motion, i.e. it has constant length and behaves like a rod).

Hint: the motion is more easily understood if one looks at it as motion of the center of mass (of m_1 and m_2) plus motion of m_1 and m_2 about the center of mass. To this purpose use the equation of motions of the system in the form of Eqs. (1.22) and (1.26) of your textbook, i.e. $\dot{\mathbf{P}} = \mathbf{F}^{(\mathbf{e})}$ and $\dot{\mathbf{L}} = \mathbf{N}^{(\mathbf{e})}$, where the only external force acting on m_1 and m_2 is the force of gravity.

If you are familiar with the treatment of a two body problem in terms of reduced variables, you can also apply this alternative approach to the solution of the problem.