November 9^{th} , 2015

Assignment # 10

(Graded problems are due Monday November 16^{th} , 2015)

1 Graded problems

- 1. A uniform solid cylinder of mass m, length b, and radius a is thrown up in the air; at the instant it is released (t = 0) it rotates with angular velocity $\omega = |\boldsymbol{\omega}|$ about an axis that passes through its center and the outside edge of one end.
 - (1.a) What is the torque on the cylinder about its center of mass due to gravity?
 - (1.b) Write down Euler's equations for the cylinder, and so find the frequency of rotation about the central longitudinal axis, and the frequency at which the angular velocity ω precesses about this axis in the body coordinate system.
 - (1.c) If the center of mass is initially moving straight upward at speed V_0 , find the total kinetic energy at later times t.
- 2. Investigate the motion of a symmetric top for the case in which the axis of rotation is vertical. Discuss the condition under which the motion is either stable or unstable. Sketch the effective potential $V(\theta)$ for the two cases and point out the features of these curves that determine if the motion is stable. If the top is set to spinning in the stable configuration, what is the effect as friction gradually reduces the value of ω_3 .
- **3.** A rigid body is made up of eight equal masses m at the corners of a wire frame with dimensions $2l(x) \times 2l(y) \times 4l(z)$. Take a body coordinate system with origin at the body center of mass. Imagine that the body is rotating with an angular velocity $\boldsymbol{\omega}$ that goes through a corner of the wire frame.
 - (3.a) If this angular velocity is constant, what happens to L in the body frame? How does L move in the fixed frame?
 - (3.b) Find the torque (expressed in the body system) required to maintain the given angular velocity ω .
- 4. A square sheet is constrained to rotate with an angular velocity ω about an axis passing through its center and making an angle α with the axis through the center of mass and normal to the sheet (i.e. its axis of symmetry). At the instant the axis of rotation lies in the plane determined by the axis of symmetry and a diagonal, the body is released. Find the rate at which the axis of symmetry precesses about the constant direction of the angular momentum.