PHY 5246: Theoretical Dynamics, Fall 2015

November 16^{th}, 2015

Assignment # 11

(Graded problems are due Monday November 23^{rd}, 2015)

1 Graded problems

1. Consider a particle in a central force field.

1.a) Obtain the Hamiltonian and the canonical equations of motion.

1.b) Take two of the initial conditions to be $p_\phi(0) = 0$ and $\phi(0) = 0$. Discuss the resulting simplification of the canonical equations.

1.c) Consider now an attractive force of magnitude k/r^2: use plane polar coordinates and find Hamilton equations of motion.

2. A particle of mass m moves in one dimension under the influence of a force

$$F(x,t) = \frac{k}{x^2}e^{-t/\tau} \ ,$$

where k and τ are positive constants.

2.a) Compute the Lagrangian and Hamiltonian functions.

2.b) Compare the Hamiltonian and the total energy, and discuss the conservation of energy for the system.

3. The Lagrangian for a particle of mass m and electric charge q moving under the influence of a magnetic (but not electric) field is given by:

$$L(\mathbf{r},\dot{\mathbf{r}}) = \frac{1}{2}m\dot{\mathbf{r}}^2 + \frac{q}{c}\dot{\mathbf{r}}\cdot\mathbf{A}(\mathbf{r}) \ ,$$

where $\mathbf{A}(\mathbf{r})$ is the vector potential. Assume that the magnetic field is constant and given by $\mathbf{B}(\mathbf{r}) = B_0\hat{\mathbf{z}}$.

3.a) Show that for such a constant magnetic field the vector potential can be written in the form $\mathbf{A}(\mathbf{r}) = \frac{1}{2}(\mathbf{B}\times\mathbf{r})$. That is, show that such a vector potential satisfies: $\nabla\times\mathbf{A} = \mathbf{B}$.

3.b) Construct the Hamiltonian of the system in terms of the Cartesian coordinates and the corresponding canonical momenta of the particle.

3.c) Denoting by $\boldsymbol{\pi} = m\dot{\mathbf{r}} = m\mathbf{v}$ the *mechanical* momentum of the particle, evaluate the following Poisson brackets:

$$\{\boldsymbol{\pi}_x, \boldsymbol{\pi}_y\} \ , \ \{\boldsymbol{\pi}_y, \boldsymbol{\pi}_z\} \ , \ \{\boldsymbol{\pi}_z, \boldsymbol{\pi}_x\} \ .$$

3.d) By re-expressing the Hamiltonian of part **3.b)** in terms of the mechanical momentum of the particle, and using the results derived in part **3.c)**, obtain the most general solution for $\boldsymbol{\pi}(t)$ by using the Poisson's equation:

$$\frac{d\boldsymbol{\pi}}{dt} = \{\mathrm{H},\boldsymbol{\pi}\} \ ,$$

where (using Landau's convention),

$$\{f,g\} = \sum_k \left(\frac{\partial f}{\partial p_k}\frac{\partial g}{\partial q_k} - \frac{\partial f}{\partial q_k}\frac{\partial g}{\partial p_k} \right) \ .$$

Interpret your results on the basis of a *conventional* (Newton's second Law plus Lorentz force) approach.

2 Non-graded suggested problems

4. Chapter 8, Problem 2 of Goldstein's book.

5. Chapter 8, Problem 7 of Goldstein's book.

6. Chapter 8, Problem 13 of Goldstein's book.

7. Chapter 8, Problem 20 of Goldstein's book.

8. Take any problem solved using Euler-Lagrange equations, derive the corresponding set of Hamilton equations, and show that you can put them in a form equivalent to the Euler-Lagrange ones. Ask yourself questions like: does the system admit constants of motion, how do you use them in finding the dynamical evolution of a system, which formalism (Lagrangian vs. Hamiltonian) better suits which problem, etc.