PHY 5246: Theoretical Dynamics, Fall 2015

November 23^{rd}, 2015

Assignment # 12

(Graded problems are due Wednesday December 2^{nd}, 2015)

1 Graded problems

1. 1.a) For a one-dimensional system with Hamiltonian

$$H = \frac{p^2}{2} - \frac{1}{2q^2} \, ,$$

show that there is a constant of motion given by

$$D = \frac{pq}{2} - Ht \ .$$

1.b) As a generalization of part **1.a)**, show that for a motion in a plane with the Hamiltonian

$$H = |\mathbf{p}|^n - ar^{-n} \ ,$$

where $\mathbf{p}$ is the vector of the momenta conjugate to the Cartesian cordinates, there is a constant of motion given by

$$D = \frac{\mathbf{p} \cdot \mathbf{r}}{n} - Ht \ .$$

1.c) The transformation $Q = \lambda q$, $p = \lambda P$ is obviously canonical. However, the same transformation with t time dilatation, $Q = \lambda q$, $p = \lambda P$, $t' = \lambda^2 t$, is not. Show that, however, the equations of motion for q and p for the Hamiltonian in part **1.a)** are invariant under this transformation. The constant of motion D is said to be associated with this invariance.

2. Given a system with Hamiltonian

$$H = \frac{1}{2}\left(\frac{1}{q^2} + p^2 q^4\right) \ ,$$

2.a) find the equation of motion for q;

2.b) find a canonical transformation that reduces H to the Hamiltonian of a harmonic oscillator. Show that the solution for the transformed variables is such that the equation of motion found in part **2.a)** is satisfied.

3. 3.a) Show that the transformation

$$Q = p + iaq \ , \quad P = \frac{p - iaq}{2ia} \ ,$$

with a a real constant, is canonical and find a generating function.

3.b) Use the transformation to solve the linear harmonic oscillator problem.