PHY 5246: Theoretical Dynamics, Fall 2015

September 2^{nd}, 2015

Assignment # 2

(Graded problems are due Wednesday September 9^{th}, 2015)

1 Graded problems

1. A bead of mass m slides without friction in a uniform gravitational field on a vertical circular hoop of radius R. The hoop is constrained to rotate at a fixed angular velocity ω about its vertical diameter. Let θ be the position of the bead on the hoop measured from the lowest point.

(1.a) Write down the Lagrangian $L(\theta, \dot{\theta})$.

(1.b) Find how the equilibrium values of θ depends on ω. Which are stable, which unstable?

(1.c) Find the frequencies of small oscillations about the stable equilibrium positions. Say something concerning the motion about the only stable equilibrium point when $\omega = \sqrt{g/R}$.

2. Consider a stretchable plane pendulum, that is, a mass m suspended from a spring of spring constant k and unstretched length l, constrained to move in a vertical plane. Write down the Lagrangian and obtain the Euler-Lagrange equations. Interpret the form of the equations in terms of the forces acting on the system.

3. Two points of mass m are joined by a rigid weightless rod of length l, the center of which is constrained to move on a circle of radius a. Express the kinetic energy in terms of generalized coordinates.

4. Two mass points of mass m_1 and m_2 are connected by a string passing through a hole in a smooth table so that m_1 rests on the table surface and m_2 hangs suspended. Assuming m_2 moves only in a vertical line, what are the generalized coordinates for the system? Write the Lagrange equations for the system and, if possible, discuss the physical significance each one of them might have. Reduce the problem to a single second-order differential equation and obtain a first integral of the equation. What is its physical significance? (Consider the motion only until m_1 reaches the hole.)

2 Non-graded suggested problems

Any problem in Chapter 1 of Goldstein's book will be a very good exercise. In particular solutions will be posted for Problems 15, 18, and 22.