PHY 5246: Theoretical Dynamics, Fall 2015

September 9^{th}, 2015

Assignment # 3

(Graded problems are due Wednesday September 16^{th}, 2015)

1 Graded problems

1. A particle slides on the inside surface of a frictionless cone subject to the action of the gravitational force. The cone is fixed with its tip on the ground and its axis vertical. Let the half-angle at the tip be α, let r be the distance from the particle to the axis of the cone, and let θ be the angle around the cone.

(1.a) Find the equations of motion.

(1.b) If the particle moves in a circle of radius r_0, what is the frequency, ω, of the motion? If the particle is perturbed slightly from this circular motion, what is the frequency, Ω, of the oscillations about the radius r_0? Under what conditions does $\Omega = \omega$?

2. A block of mass m is held motionless on a frictionless plane of mass M and angle of inclination θ. The inclined plane rests on a frictionless horizontal surface. The block is released and start sliding under the action of the force of gravity. What is the horizontal acceleration of the inclined plane? (Try solving this problem using Newtonian mechanics, i.e. $\mathbf{F} = m\mathbf{a}$. You will have a greater appreciation for the Lagrangian method!)

3. A particle is subject to a potential $V(x) = -Fx$ where F is a constant. The particle travels from $x = 0$ to $x = a$ in a time interval t_0. Assume the motion of the particle can be expressed in the form $x(t) = A + Bt + Ct^2$. Find the values of A, B, and C such that the action is a minimum.

4. Suppose a particle moves in space subject to a conservative potential $V(\mathbf{r})$ but is constrained to always move on a surface whose equation is $\sigma(\mathbf{r}, t) = 0$ (where the explicit dependence on t indicates that the surface may be moving). The instantaneous force of constraint is taken as always perpendicular to the surface. Show analytically that the energy of the particle is not conserved if the surface moves in time. What physically is the reason for non-conservation of the energy under this circumstance?

2 Non-graded suggested problems

5. Goldstein's book, Chapter 2, Problem 19.

6. Goldstein's book, Chapter 2, Problem 6.