PHY 5246: Theoretical Dynamics, Fall 2015

September 23^{rd}, 2015
Assignment # 5
(Graded problems are due Wednesday September 30^{th}, 2015)

1 Graded problems

1. Consider a particle that moves in a logarithmic spiral orbit given by $r = ke^{\alpha\theta}$, where k and α are constants.

(1.a) Find the force law that allows the particle to move in this orbit.

(1.b) Determine $r(t)$ and $\theta(t)$.

(1.c) What is the total energy of the orbit?

2. A particle of mass m moves in a potential given by $V(r) = \beta r^k$, where β and k are constants. Let the angular momentum be l.

(2.a) Find the radius r_0 of the circular orbit.

(2.b) If the particle is given a tiny kick so that the radius oscillates around r_0, find the frequency, ω_r, of these small oscillations in r.

(2.c) What is the ratio of the frequency ω_r to the frequency of the (nearly) circular motion, $\omega_\theta = \dot{\theta}$? Describe the cases: $k = -1, 2, 7, -\frac{7}{4}$, for which the ratio ω_r/ω_θ is rational, that is, for which the path of the nearly circular motion closes back on itself. Can you roughly plot the orbits for these four cases?

3. Two particles move about each other in circular orbits under the influence of gravitational forces, with a period τ. Their motion is suddenly stopped at a given instant of time, and they are released and allowed to fall into each other. Prove that they collide after a time $\frac{\tau}{4\sqrt{2}}$.

2 Non-graded suggested problems

4. A particle moves in a force field described by

$$F(r) = -\frac{k}{r^2}\exp\left(-\frac{r}{a}\right) \quad ,$$

where k and a are positive.

(4.a) Write the equations of motion and reduce them to the equivalent one-dimensional problem. Use the effective potential to discuss the qualitative nature of the orbits for different values of the energy and the angular momentum.

(4.b) Show that if the orbit is nearly circular, the apsides will advance approximately by $\pi\rho/a$ per revolution, where ρ is the radius of the circular orbit.