PHY 5246: Theoretical Dynamics, Fall 2015

September 30^{th}, 2015
Assignment # 6
(Graded problems are due Friday October 7^{th}, 2015)

1 Graded problems

1. Consider an attractive $1/r$ potential and show that:

1.a) for circular and parabolic orbits having the same angular momentum, the perihelion distance of the parabola is one-half the radius of the circle;

1.b) the speed of a particle at any point in a parabolic orbit is $\sqrt{2}$ times the speed in a circular orbit passing through the same point.

2. Discuss the motion of a particle in a central inverse-square law force field for a super-imposed force whose magnitude is inversely proportional to the cube of the distance from the particle to the center of force, that is

$$F(r) = -\frac{k}{r^2} - \frac{\lambda}{r^3} \quad k, \lambda > 0 \ .$$

Show that the motion is described by a precessing ellipse. Consider the cases $\lambda < l^2/m$, $\lambda = l^2/m$, and $\lambda > l^2/m$.

3. Consider a particle describing a circular orbit under the influence of an attractive central force directed toward a point in the circle.

3.a) Show that the force varies as the inverse-fifth power of the distance.

3.b) Show that for the orbit described the total energy is zero.

3.c) Find the period of the motion.

3.d) Find $\dot{x}$, $\dot{y}$, and v as a function of the angle around the circle and show that all three quantities are infinite as the particle goes through the center of force.

4. A particle is moving in a potential

$$V(r) = -\frac{C}{3r^3} \quad (C > 0) \ .$$

4.a) Given l (angular momentum), find the maximum value of the effective potential.

4.b) Let the particle come in from infinity with speed v_0 and impact parameter b. In terms of C, m, and v_0, what is the largest value of b (call it b_{max}) for which the particle is captured by the potential? In other words, what is the *cross section* for capture, πb_{max}^2, for this potential?

2 Non-graded suggested problems

5. Chapter 3, Problem 10 of Goldstein's book.