PHY 5246: Theoretical Dynamics, Fall 2015

October 7^{th}, 2015
Assignment # 7
(Graded problems are due Friday October 14^{th}, 2015)

1 Graded problems

1. A particle of mass m travels in a hyperbolic orbit past a mass M, whose position is assumed to be fixed. The speed at infinity is v_0, and the impact parameter is b.

(1.a) Show that the angle through which the particle is deflected is

$$\Theta = \pi - 2\tan^{-1}(\gamma b) \quad \Rightarrow \quad b = \frac{1}{\gamma}\cot\left(\frac{\Theta}{2}\right) \quad ,$$

where $\gamma \equiv v_0^2/(GM)$.

(1.b) Let $d\sigma$ be the cross-sectional area (measured when the particle is initially at infinity) that gets deflected into a solid angle of size $d\Omega$ at angle Θ (this quantity is called *differential cross section*). Show that

$$\frac{d\sigma}{d\Omega} = \frac{1}{4\gamma^2\sin^4(\Theta/2)} \quad .$$

(1.c) Consider the case of *backward scattering*, i.e. $\Theta \approx 180°$. What can you tell in the limiting cases of small v_0 ($v_0 \to 0$) and large v_0 ($v_0 \to \infty$)? Explain your results.

(1.d) Consider the case of negligible deflection, i.e. $\Theta \approx 0°$. Does it make sense that $\sigma \approx \infty$ and why? How should the potential behave in order not to generate an infinite cross section?

(1.e) Show that in case you replace the gravitational force with the electrostatic one (Coulomb interaction between pointlike charges) you get Rutherford-scattering differential cross section:

$$\frac{d\sigma}{d\Omega} = \frac{K^2 q_1^2 q_2^2}{16E^2\sin^4(\Theta/2)} \quad .$$

2. Examine the scattering produced by a repulsive central force $f = kr^{-3}$. Show that the differential cross section is given by

$$\sigma(\Theta)d\Theta = \frac{k}{2E}\frac{(1-x)dx}{x^2(2-x)^2\sin(\pi x)} \quad ,$$

where $x = \Theta/\pi$ and E is the energy.

3. Show that the angle of scattering in the laboratory system, θ, is related to the energy before scattering (E_0) and the energy after scattering (E_1) according to the equation

$$\cos\theta = \frac{m_1 + m_2}{2m_1}\sqrt{\frac{E_1}{E_0}} + \frac{m_1 - m_2}{2m_1}\sqrt{\frac{E_0}{E_1}} + \frac{m_2 Q}{2m_1\sqrt{E_0 E_1}} \ ,$$

where Q is the so-called $Q - value$ of the collision.

2 Non-graded suggested problems

5. Chapter 3, Problem 32 of Goldstein's book.