PHY 5246: Theoretical Dynamics, Fall 2015

October 21^{st}, 2015
Assignment # 9
(Graded problems are due Wednesday October 28^{th}, 2015)

1 Graded problems

1. Find the frequency of small oscillations for a thin homogeneous plate if the motion takes place in the plane of the plate and if the plate has the shape of an equilateral triangle and is suspended **(a)** from the midpoint of one side and **(b)** from the apex.

2. A homogeneous cube, each edge of which has a length l, is initially in a position of unstable equilibrium with one edge in contact with a horizontal plane. The cube is then given a small displacement and allowed to fall. Show that the angular velocity of the cube when one face strikes the plane is given by

$$\omega^2 = A\,\frac{g}{l}\,(\sqrt{2}-1) \ ,$$

where $A = 3/2$ is the edge cannot slide on the plane and where $A = 12/5$ if sliding can occur without friction.

3. A uniform right circular cone of height h, half-angle α, and density ρ rolls on its side without slipping on a uniform horizontal plane in such a manner that it returns to its original position in a time τ. Find expressions for the kinetic energy and the components of the angular momentum of the cone.

4. A square sheet is constrained to rotate with an angular velocity ω about an axis passing through its center and making an angle α with the axis through the center of mass and normal to the sheet (i.e. its axis of symmetry). At the instant the axis of rotation lies in the plane determined by the axis of symmetry and a diagonal, the body is released. Find the rate at which the axis of symmetry precesses about the constant direction of the angular momentum.

2 Non-graded suggested problems

5. Show that none of the principal moments of inertia can exceed the sum of the other two.

6. Calculate the moments of inertia I_1, I_2, and I_3 for a homogeneous cone of mass M whose height is h and whose base has a radius R. Choose the x_3-axis along the axis of the cone. Choose the origin at the apex of the cone, and calculate the elements of the inertia tensor. Then make a transformation such that the center of mass of the cone becomes the origin, and find the principal moments of inertia.

7. Consider a thin disk composed of two homogeneous halves connected along a diameter of the disk. If one half has density ρ and the other has density 2ρ, find the expression for the Lagrangian when the disk rolls without slipping along a horizontal surface (the rotation takes place in the plane of the disk).