

PHY 5667 : Quantum Field Theory A, Fall 2015

August 27th, 2015

Assignment # 1

(due Thursday September 3rd, 2015)

1. Write down the Lagrangian of a relativistic free particle and show that, in the limit $v/c \rightarrow 0$ (where v is the magnitude of the particle velocity and c is the velocity of light) it reduces to the Lagrangian of a non-relativistic free particle modulus a constant. Explain why this does not affect the form of the non-relativistic equations of motion.
2. It is often possible to derive a field theory as the limit of a discrete system. Perhaps the simplest example is an infinite system of point masses, m , separated by springs of spring constant k and equilibrium length a . Let η_i be the displacement from equilibrium of the i th point mass. Derive the exact Lagrangian and the Lagrange equations for this system. Then consider the limit

$$m, a \rightarrow 0 \quad , \quad k \rightarrow \infty \quad , \quad \mu = m/a \text{ and } Y = ka \text{ fixed .}$$

Replacing η_i by a smooth function $\eta(x, t)$, show that in this limit the Lagrangian may be written in the density form

$$L = \int dx \frac{1}{2} \left[\mu \left(\frac{\partial \eta}{\partial t} \right)^2 - Y \left(\frac{\partial \eta}{\partial x} \right)^2 \right]$$

and write down the corresponding (partial differential) Lagrange equations.

3. The electromagnetic field may be specified by a vector $A^\mu(\mathbf{x}, t)$, in terms of which the Lagrange density of the field is

$$\mathcal{L}(x) = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} \quad ,$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \quad .$$

Derive the Lagrange equations for this system, and express them in terms of the free-space field strengths $\mathbf{E} = -\partial_0 \mathbf{A} - \nabla A_0$ and $\mathbf{B} = \nabla \times \mathbf{A}$. How many of Maxwell's equations does this give, and why are the others also satisfied?