PHY 5667 : Quantum Field Theory A, Fall 2015

September 24^{th}, 2015

Assignment # 4

(due Thursday October 8^{th}, 2015)

1. Under a Lorentz transformation (Λ) Dirac (and Majorana) fields transform as,

$$\psi'(x) = D(\Lambda)\psi(\Lambda^{-1}x) \ ,$$

where, denoting by $S^{\mu\nu}$ the generators of Lorentz transformations in the Dirac spinor representation, $D(\Lambda)$ for an infinitesimal transformation can be written as,

$$D(\Lambda) = 1 + \frac{i}{2}\delta\omega_{\mu\nu}S^{\mu\nu} \ .$$

1.a Find the form of the generators $S^{\mu\nu}$.

1.b Find in this representation the explicit form of a finite rotation by an angle θ about the z axis.

1.c Find in this representation the explicit form of a finite boost by rapidity η in the z direction.

2. Show that, for spinors $u^s(p)$ and $v^s(p)$,

$$\sum_{s=1,2} u^s(p)\bar{u}^s(p) = \gamma \cdot p + m \ ,$$

and

$$\sum_{s=1,2} v^s(p)\bar{v}^s(p) = \gamma \cdot p - m \ .$$

3. Prove the *Gordon identity*:

$$\bar{u}(p')\gamma^\mu u(p) = \bar{u}(p')\left[\frac{(p'+p)^\mu}{2m} + i\frac{\sigma^{\mu\nu}q_\nu}{2m}\right]u(p) \ ,$$

where $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$ and $q = (p' - p)$.

4. Problem 3.4 of Peskin and Schroeder's book.

5. Problem 3.6 of Peskin and Schroeder's book.