PHY 5667 : Quantum Field Theory A, Fall 2015

October 8^{th}, 2015
Assignment # 5
(due Thursday October 22^{nd}, 2015)

1. Consider the action of *parity* (P), *time-reversal* (T), and *charge-conjugation* operators on spinor quantum fields as defined in Sec. 4.2.3 of Maggiore's book or in Sec. 3.6 of Peskin and Schoeder's book, and find the transformation properties of the following spinor bilinear: $\bar{\psi}\psi$, $i\bar{\psi}\gamma^5\psi$, $\bar{\psi}\gamma^\mu\psi$, $\bar{\psi}\gamma^\mu\gamma^5\psi$, and $\bar{\psi}\sigma^{\mu\nu}\psi$.

2. Following the example of the scalar-field (Feynman) propagator that we discussed in detail in class, calculate the Feynman propagator for Dirac spinor fields

$$S_F(x-y) \equiv < 0|T\psi(x)\bar{\psi}(y)|0 > \ ,$$

and show that

$$S_F(x-y) = \int \frac{d^4p}{(2\pi)^4}\tilde{S}_F(p)e^{-ip(x-y)} \ ,$$

where

$$\tilde{S}_F(p) = \frac{i(\not{p}+m)}{p^2-m^2+i\epsilon} \ ,$$

while the meaning of the $i\epsilon$ term in the denominator should be clear from the discussion of the scalar-field propagator.

3. Considered a quantum system of vector fields described by a Maxwell Lagrangian and quantized using radiation gauge ($A_0 = 0$, $\nabla \cdot \mathbf{A} = 0$). Calculate the explicit expression of the Hamiltonian (H) and momentum ($\mathbf{P}$) operators.

4. Consider the following Lagrangian,

$$L = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}m^2A^\mu A_\mu \ ,$$

with $m \neq 0$, also known as *Proca Lagrangian.*

4.a) Verify that this theory is not gauge invariant.

4.b) Derive the equations of motion and show that they are of the form

$$(\partial^2 + m^2)A^\mu = 0 \ , \ \ \partial_\mu A^\mu = 0 \ .$$

Write the most general solution as a superposition of momentum-space modes.

4.c) Perform the canonical quantization and verify that this theory describes a massive spin-1 particle (a system of).

5. Explain why the action functional has the same dimensions of $\hbar$, and therefore is dimensionless in units of $c = \hbar = 1$. Based on this:

5.a) find the (mass) dimensions of scalar, spinor, and vector fields in $d = 4$ space-time dimensions;

5.b) repeat the same analysis in d space-time dimensions, i.e. when the action is written as $S = \int d^d x \mathcal{L}$ and $\mathcal{L}$ is the same as in $d = 4$.