PHY 5667 : Quantum Field Theory A, Fall 2015

November 10^{th}, 2015
Assignment # 7
(due Thursday November 19^{th}, 2015)

1. Consider a real scalar field theory with $\mathcal{H}_{\text{int}} = \frac{\lambda}{4!}\phi^4$.

- **1.a)** Derive the tree-level vertex interaction that defines the vertex Feynman rule for this theory in momentum space.
- **1.b)** Given a generic diagram with E external legs, V vertices, and P propagators, what is the relation among E, V, and P for the diagram to be fully connected?
- **1.c)** Draw all the fully connected diagrams with $1 \leq E \leq 4$ and $0 \leq V \leq 2$, and find their symmetry factors.

2. Repeat the same exercise for the case of a complex scalar field with $\mathcal{H}_{\text{int}} = \frac{\lambda}{4}(\phi^\dagger\phi)^2$. Remember that there are now two kinds of particles (which we can think as positively and negatively charged), and that your rules must have a way to distinguish among them. *Hint*: it seems like you need two kinds of *arrows* to represent the flow of momentum and charge separately. Try to find a more elegant approach that only needs one kind of arrows.

3. Consider an interacting scalar field theory with $H_{\text{int}} = \frac{\lambda}{4}\phi^4$. Sketch the diagrams contributing to the invariant scattering matrix element $\mathcal{M}(p_1, p_2 \to p_3 p_4)$ at $O(\lambda^2)$ and write the corresponding explicit expression in momentum space.

4. Consider the following Lagrangian involving two real scalar fields Φ and ϕ:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\Phi)^2 - \frac{1}{2}M^2\Phi^2 + \frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}m^2\phi^2 - \mu\Phi\phi\phi\ .$$

The last term is an interaction that allows a Φ particle to decay into two ϕ's if $M > 2m$. Assuming that this condition is met, calculate the lifetime of the Φ particle to lowest order in μ.

5. Consider the two-fermion scattering process:

$$\text{fermion}(p) + \text{fermion}(k) \to \text{fermion}(p') + \text{fermion}(k')$$

in the context of the Yukawa theory. Calculating the differential cross section $\left(\frac{d\sigma}{d\Omega}\right)_{CM}$ and the total cross section σ at the lowest order or *tree level*.